(Optimal) Scheduling

in increasingly realistic models, with applications

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HUAWEI

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Tile-based programming

> **Tile-based programming** is a development paradigm that abstracts individual thread management into operations on discrete, local data blocks called **tiles**, allowing the compiler and run-time to automate hardware-specific optimisations like memory movement and **Tensor Core** utilisation.
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- Triton (run using an $m/BI \times n/BJ$ process grid):

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s = tl.program_id( 0 ); t = tl.program_id( 1 );
off_i = s * BI + tl.arange( 0, BI );
off_j = t * BJ + tl.arange( 0, BJ );
a_tile = tl.load( A_ptr + off_i * n + off_j );
b_tile = tl.load( B_ptr + off_i * n + off_j );
c_tile = a_tile + b_tile;
tl.store( C_ptr + off_i * n + off_j, c_tile );
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- PyPTO (call using standard Torch tensors and JIT):

```
# A, B, C: pypto.Tensor( shape, pypto.DT_FP32 );
pypto.set_vec_tile_shapes( 1, 4 );
C = A + B;
```

Tile-based programming

Main difference:

- Triton and like: user decides memory layout, tiling, blocking, and writes tiled algorithm **explicitly**;
- "Tensor-level" PyPTO: users express computation, PTO decides data layout and produces a tiled algorithm **implicitly**.

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Nevertheless, this approach pushes a **hard problem** to the framework:

- the tensor-level problem is decomposed into a fine-grained graph;
- which it schedules over multiple vector and cube cores; and
- may need to distribute over multiple dies, devices, nodes.

Scheduling in realistic models

First, assume **realistic costs**:

- compute, data movement (g), *and* latency (l)– i.e., BSP.

Assume work must be near-balanced (up to some $\epsilon > 0$).

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2022/2023:

Theorem

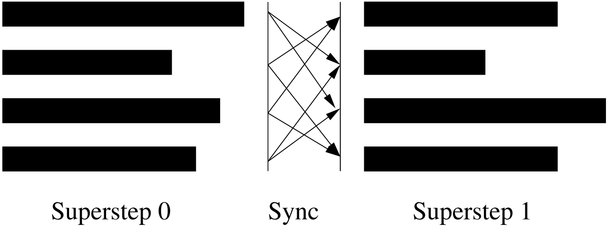
Assuming ETH, it is not possible to approximate the optimum of the ϵ-balanced hypergraph partitioning problem to an $n^{(\log\log n)^{-\delta}}$ factor in polynomial time, for some $\delta > 0$.

This already holds when the hypergraphs are DAGs.

Ref.: Partitioning Hypergraphs is Hard: Models, Inapproximability, and Applications, Pál András Papp, Georg Anegg, and Y., SPAA '23 (Theorem 4.1).

Scheduling in realistic models

Scheduling via partitioning by imposing BSP supersteps:



i.e., divide DAG into supersteps, then layer-wise ϵ-balanced partitioning.

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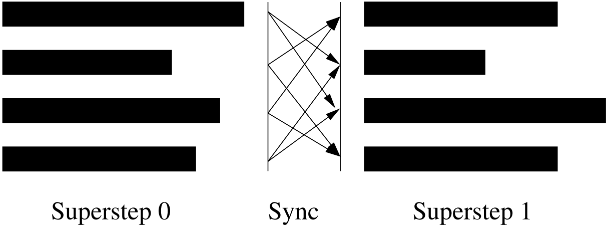

i.e., divide DAG into supersteps, then layer-wise ϵ-balanced partitioning.

Theorem

It is NP-hard to approximate the layer-wise balanced partitioning problem to any finite factor.

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When combining horizontal *and* vertical data movement:

Theorem

Multi-processor pebbling is APX-hard. (There is a constant $\epsilon > 0$ s.t. it is NP-hard to approximate to a $(1 + \epsilon)$ factor.)

Ref.: DAG Scheduling in the BSP Model, Pál András Papp, Georg Anegg, and Y., SOFSEM '25 **best paper** award. (Theorems 2–5).
Ref.: Multiprocessor Scheduling with Memory Constraints: Fundamental Properties and Finding Optimal Solutions, Pál András Papp, Toni Böhnlein, and Y., ICPP '25. (Theorem 1).

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  - these may differ, while here the DAGs are kept the same(!);
- similar for vertical data movement (NP-hard to $n^{1-\epsilon}$, $\forall\epsilon > 0$),
  - with and without **partial computations**.

Ref.: DAG Scheduling in the BSP Model, Papp, Anegg, Y., SOFSEM '25, **best paper award**
Ref.: The Impact of Partial Computations on the Red-Blue Pebble Game, Papp, Sobczyk, Y, SPAA '25
Ref.: Replication in Graph Partitioning and Scheduling Problems, Papp, Böhnlein, Y. (submitted)

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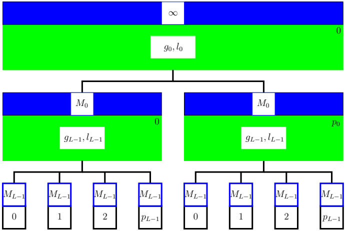
Modern systems: pervasively **NUMA**. Consider Multi-BSP (Valiant '08, '11):



Theorem

Given an optimal BSP solution to ϵ-balanced hypergraph partitioning and a two-level Multi-BSP ($d = 2$). Optimal hierarchical assignment is NP-hard for $p_2 > 2$, and results in a g_1-approximation (up to $\frac{p_1 - 1}{p_1} g_1$).

(See paper for $d > 2$).

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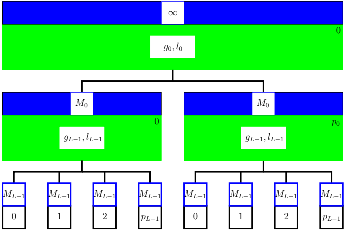

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(See paper for $d > 2$). "Pure" scheduling or MPP hardness for $d \geq 2$: TBD.

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Practical scheduling in realistic models

We show three types of **experimental results**:

1) **model-based** evaluation– i.e., (Multi-)BSP cost;
2) **real-world** evaluation on sparse triangular solves– i.e., time;
3) summary of gains for PyPTO– i.e., speedups.

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Algorithms evaluated:

- optimal schedules via **ILP formulations** (COPT);
- ILP formulations for iterative refinement
  - two supersteps, comm. optimisation;
- heuristic **initialisers**: greedy BSP (g_k, l_k), growLocal (l), Sarkar;
- **local search**: hill climbing (g_k, l_k), Kernighan-Lin (g_k, l_k);
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- also available: Ubuntu and RedHat packages, OSP-as-a-service.

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We *can* compute practical **optimal** schedules in realistic models:

- but only for **small DAGs**: hundreds of nodes, at most.
  - CBC, improves with the use of commercial ILP solvers.
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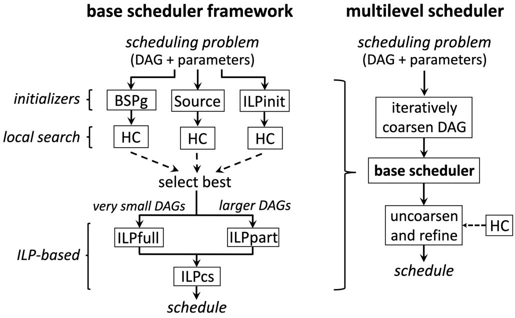

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Model-based evaluation, BSP and NUMA cost reductions.

- Diverse computational dataset from the HyperDAG database;
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  - uniform scheduling: $0.56\times$, $0.76\times$;
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- $d = 4$, $p = 16$, $g = 1$, $l = 5$, geomean:
  - uniform scheduling: $0.57\times$, $0.70\times$;
  - NUMA-aware scheduling: $0.29\times$, $0.42\times$;
  - multi-level scheduling: $0.13\times$, $0.21\times$ (!!)
- The SPAA paper contains further experiments.

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Model-based evaluation, multi-processor pebbling, ILP solvers.

- two-level with memory constraints, 40-80 node (hyper)DAGs, $p = 4$, $g = 1$, $l = 10$ (small problems), and $M = 3\sum_{v \in V} m(v)$:
  - 0.77× the cost of greedy BSP & clairvoyant LRU;
  - 0.88× the cost of **optimal** BSP & clarivoyant LRU;
  - 0.66× the cost of Cilk & regular LRU (**online** SotA).

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- larger HyperDAGs up to 264-464 nodes, $M = 5\sum_v m(v)$:
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  - **divide-and-conquer** ILPs: 0.66–1.13× the cost of SotA.
- the above results employ **recomputation** when useful;
  - max. penalty if disallowed: 1.4× optimality.

ILP formulations solved using **commercial solvers** (COPT).

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Sparse triangular solve by DAG scheduling, Intel CPU, 22 cores:

Dataset	GL	Funnel+GL	SpMP	HDagg
SuiteSparse	10.8	10.2	7.6	3.3
SuiteSparse (permuted)	15.9	15.4	9.4	9.0
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All of this (optimal & heuristics) are available freely, Apache 2.0:

- github.com/Algebraic-Programming/OneStopParallel

Ref.: Efficient Parallel Scheduling for Sparse Triangular Solvers, Toni Böhnlein, Pál András Papp, Raphael S. Steiner, Christos K. Matzoros, Y., arXiv: 2503.05408; pre-print (June 2025).

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- algorithm & parameter (g, l) tuning, mod pre-scheduling passes.

Code: Böhnlein, Steiner, Matzoros, De Vita, Papp, Y., '25-'26:

- `https://gitcode.com/cann/pypto/merge_requests/905`
- `https://github.com/Algebraic-Programming/OneStopParallel`

Ref.: Partitioning parallel programs for macro-dataflow, V. Sarkar and J. Hennessy, Proc. ACM Conference on LISP and Functional Programming (1986).

Conclusion

We show:

- **hardness** of scheduling under realistic costing (g, l, and NUMA);
- ILP formulations for **optimal scheduling** (horizontal & vertical);
- heuristics (local search, refinement), multi-level, divide & conquer;
- **significant practical gains** over four systems:
  - 2.2–4.2, 1.4–1.7$\times$ faster SpTrsv vs. HDagg, SpMP;
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- maxBSP (Valiant '90) and **SSP** with arbitrary staleness k
  - additional $\approx$ 20% gain for SpTrsv ($k = 1$), paper soon;
- Optimal Multi-BSP **partitioning**, for training & online inference:
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It's open!

Open source, Apache 2.0, welcome to try, use, and collaborate!

- `https://github.com/Algebraic-Programming`
- `https://algebraic-programming.github.io`

Publications:

- Y., Di Nardo, Nash, Suijlen: A C++ GraphBLAS (2020);
- Mastoras, Anagnostidis, Y.: Design and implementation for nonblocking execution in GraphBLAS: tradeoffs and performance, ACM TACO (2023);
- Scolari, Y.: Effective implementation of the High Performance Conjugate Gradient benchmark on ALP/GraphBLAS, IPDPSW (GrAPL 2023);
- Spampinato, Jelovina, Zhuang, Y.: Towards Structured Algebraic Programming, ACM ARRAY (2023);
- Papp, Anegg, Y.: Partitioning Hypergraphs is Hard: Models, Inapproximability, and Applications, ACM SPAA (2023);
- Papp, Anegg, Karanasiou, Y.: Efficient Multi-Processor Scheduling in Increasingly Realistic Models, ACM SPAA (2024);
- Y.: Humble Heroes, Communications of Huawei Research (2024);
- Pasadakis, Schenk, Vlačić, Y.: Nonlinear spectral clustering with C++ GraphBLAS, extended abstract, IEEE HPEC (2023, **outstanding short paper**);
- Papp, Anegg, Y.: DAG scheduling in the BSP model, SOFSEM (2025, **best paper**);
- Niu, Meyer, Pasadakis, Y., Schenk: Incremental Sparse Tensor Format for Maximizing Efficiency in Tensor–Vector Multiplications, IEEE Cluster (2025, **best poster**);
- Martinez-Ferrer, Y., Beltran: Distributed and heterogeneous tensor-vector contraction algorithms for high performance computing, Elsevier FGCS (2025);
- Papp, Sobczyk, Y.: The Impact of Partial Computations on the Red-Blue Pebble Game, ACM SPAA (2025);
- Papp, Böhnlein, Y.: Replication in Graph Partitioning and Scheduling Problems (in submission);
- Mastoras, Y.: Efficient handling of sparse vectors for parallel nonblocking execution in GraphBLAS, GraphSys at EuroPar (2025, to appear).

Humble and Hero Programming

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Many existing software, many workload domains, many architectures:

- **re-define compiler, library, and application boundaries**

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Three ALP concepts: algebraic *containers*, *structures*, and *primitives*.

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grb::Vector< std::pair< int, double > > pairs( n );
grb::Vector< bool > s( n, 1 );       // nz cap: one
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std::cout << "s␣has␣ID␣" << grb::getID( s ) << "\n";
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   grb::operators::mul< double >,
   grb::identities::zero, grb::identities::one
> mySemiring;
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Algebraic primitives operate on algebraic containers:

- grb :: set(x, 1.0); // $x_i = 1,\ \forall i$
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- grb::mxv(y, A, x, mySemiring); // y += Ax; in-place

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All primitives except 'getters' such as grb :: { size , nrows, nnz, capacity }:

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All primitives except 'getters' such as grb::{ size ,nrows,nnz, capacity }:

- allow input & output **masks**, **descriptors**, and **phase** arguments;
- return **error codes** such as for mismatching dimensions.

Algebraic type traits

Algebraic type traits: compile-time introspection of algebraic info

- grb :: is_associative< Operator >::value, `true` iff $(a \odot b) \odot c = a \odot (b \odot c)$;
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These are all **compile-time constant** (through C++11 constexpr):

- similar to the standard C++11 *type traits*.

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Algebraic type traits help

- detect programmer errors,
- decide which optimisations are applicable, and
- reject expressions without recipe for auto-parallelisation.

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For example, **algebraic type traits prevent**

- creating a monoid from non-associative operator;
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- reducing a sparse vector to a scalar without a monoid structure.
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  - if S is **associative** and has an **identity**;
- reorder computation to improve cache hit rates
  - if S is **commutative**;
- replace primitives with cheaper ones:
  - `grb::eWiseApply( z, x, x, S );` sets $z_i = x_i \odot x_i$;
  - $\odot = \min$? Replace eWiseApply with `grb::set( z, x );`!
  - every time S is **idempotent**;
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- ...

These optimisations are applied at compile-time,

without requiring programmer knowledge or intervention.

Ex.: Y & Bisseling '10; Y & Roose '14; Y, Bisseling, Roose, Meerbergen '14; Y, Roose, Meerbergen '14; ...

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Every backend defines **performance semantics**:

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- guide programmers to express the best possible algorithm;
- gauge scalability: compute resources vs. problem size;
- expose trade-off opportunities: e.g., speed vs. memory;
- automatic choice of algorithms and backends.

Performance semantics

Every ALP program can be **systematically costed**:

Primitive	Work	Ops	Data movement	Reductions
setElement(x, y, i)	1	-	1	no
set(x, y)	$\min\{n, nz_x + nz_y\}$	-	$nz_x + nz_y$ or $n + nz_y$	no
clear(x)	nz_x	-	nz_x	no
apply$(z, x, y, \odot/M)$	$\min\{n, nz_x + nz_y\}$	$nz_{x\cap y}$	$2\min\{n, nz_x + nz_y\} + nz_{x\cup y}$	no
foldl$(y, x, \odot/M)$ foldr$(x, y, \odot/M)$	nz_x	$nz_{x\cap y}$	$2nz_x$	no
foldl$(y, \alpha, \odot/M)$ foldr$(\alpha, y, \odot/M)$ foldl(α, y, M) foldr(y, α, M)	nz_y	nz_y	nz_y	no yes
mul(z, x, y, R)	$\min\{nz_x, nz_y\}$	$nz_{x\cap y}$	$2\min\{nz_x, nz_y\} + nz_{x\cap y}$	no
dot$(z, x, y, (M, \odot))$ dot(z, x, y, R)	n $\min\{nz_x, nz_y\}$	$2n$ $2 \cdot nz_{x\cap y}$	$2n$ $2\min\{nz_x, nz_y\}$	yes

Level-1 primitives and their costs, excluding masking. Similar tables exist for level-2 and level-3 primitives.

Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by Y., D. Di Nardo, J. M. Nash, and W. J. Suijlen (2020).

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Capacities are **optional** during container construction:

```
grb::Vector< bool > s( n, 1 );
grb::Matrix< void > L( n, n, nz );
```

Out of memory errors throw exceptions; primitives return error codes.

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Capacities:

- are **lower bounds**; grb::capacity(s) ≥ 1;
- may **increase** through grb::resize, updates memory use semantics;
- Any request to decrease capacity thus **may be ignored**.

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Selecting the **sequential** auto-vectorising backend:

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```
grbcxx --backend reference_omp -o myProgram myProgram.cpp
grbrun -b reference_omp ./myProgram datasets/west0497.mtx
```

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Selecting the **1D distributed-memory parallel** backend (4 nodes):

```
grbcxx -b bsp1d -o myProgram myProgram.cpp
grbrun -b bsp1d -np 4 ./myProgram datasets/west0497.mtx
```

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Selecting the **hybrid shared and dist. parallel** backend (10 nodes):

```
grbcxx -b hybrid -o myProgram myProgram.cpp
grbrun -b hybrid -np 10 ./myProgram datasets/west0497.mtx
```

Performance

For the nonblocking backend, expect:

- speedup *up to* pipeline depth if $nz = \Theta(n)$;

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Ref.: —, "Design and implementation for nonblocking execution in GraphBLAS: tradeoffs and performance", ACM TACO, 2023.

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Similar results for PageRank and sparse deep neural network inference.

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Performance

Scale-out performance of graph algorithms, using the hybrid backend:

- Clueweb12 link matrix, approx. 978M vertices and 42.5B edges

	Ivy Bridge nodes						
	4	5	6	7	8	9	10
Input	1524	1271	1067	943	691	662	537
4-hop reachability BFS	48.8	110	54.8	99.6	83.0	74.2	23.3
20-hop reachability BFS	404	280	231	323	221	230	160
PageRank	13.3	10.3	9.68	8.00	21.0	22.9	21.6

The k-hop BFS and PageRank (PR) on Clueweb12, performance in seconds. Infiniband EDR interconnect.

Ref.: updated (ALP/GraphBLAS v0.4) results from "A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation" by Y. et al. (2020)
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  - easily extended, welcoming contributions.
- complex support, explicit *and* user-defined preconditioning.

Interface: (see also `http://albert-jan.yzelman.net/alp/v0.8-preview/`)

```
template< grb::Descriptor descriptor >
grb::RC preconditioned_conjugate_gradient(
              grb::Vector< IOType >         &x,
        const grb::Matrix< NonzeroType > &A,
        const grb::Vector< InputType >    &b,
        const std::function< grb::RC(
            grb::Vector< IOType > &,
            const grb::Vector< IOType > &
        ) > &Minv,
        const size_t max_iterations,  ResidualType tol,
              size_t &iterations,     ResidualType &residual,
```

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        grb::Vector< IOType > &,
        const grb::Vector< IOType > &
    ) > &Minv,
    const size_t max_iterations, ResidualType tol,
          size_t &iterations,     ResidualType &residual,
    grb::Vector< IOType > &buffer1, grb::Vector< IOType > &buffer2,
    grb::Vector< IOType > &buffer3, grb::Vector< IOType > &buffer4
);
```

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Transition paths ensure transparant compatibility:

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  - disadvantage: loses some ALP automatisms and performance
- CRS-compatible sparse iterative solvers:

```
double *x, *b, *a_vals; int *a_cols, *a_offs;
// ...

sparse_cg_handle_t handle;
sparse_cg_init( &handle, n, a_vals, a_cols, a_offs );

sparse_cg_solve( handle, x, b );
sparse_cg_destroy( handle );
```

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- CRS-compatible solvers with user-defined preconditioning:

```
double *x, *b, *a_vals; int *a_cols, *a_offs;
int my_preconditioner( double * out, const double * in, void * data );
// ...
sparse_cg_handle_t handle;
sparse_cg_init( &handle, n, a_vals, a_cols, a_offs );
sparse_cg_set_preconditioner( handle, my_preconditioner, data );
sparse_cg_solve( handle, x, b );
sparse_cg_destroy( handle );
```

CG: transition and embrace results, v0.8 (prelim.)

Transition vs. embrace path performance on 2-socket ARM, 96 cores:

- non-preconditioned CG, 1000 iterations, not converged;
    - vectorisation: 16 bytes, nz-indices: `uint`, non-blocking: L1.

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ALP up to $28.5\times$ faster vs. Eigen and 6.96, $2.01\times$ vs. hand-optimised.

Performance

ALP interoperability with existing (parallel) frameworks

- standard Spark/Scala interface, Spark is **not modified**;
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				Spark				Spark with ALP/GraphBLAS			
	GB	Gnz	n_ϵ	$n = 1$	$n = 10$	$n = n_\epsilon$	s/it.	$n = 1$	$n = 10$	$n = n_\epsilon$	s/it.
uk-2002	4.7	0.3	73	168.6	1373.8	>4 hrs	133.9	8.7	13.9	48.7	0.56
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- v0.8 (prelim.), **3 nodes**, uk-2002, IB EDR, dual-socket ARM:
  - 113 ms./iter, 19.2× vs. GraphX. See ALP/Spark @ GitHub

Ref.: Suijlen and Y., "Lightweight Parallel Foundations", (2019); ALP v0.8-preview and ALP/Spark (2024).

Beyond ALP/GraphBLAS

How far can we take this type of programming?

The Alps

The Alps:

- Monte Rosa,
- Matterhorn,
- Weisshorn,
- Jungfrau,
- Rothorn,
- Dom,
- ...

The ALPs

Algebraic Programming

IRs, communication layers, domain-specific languages, libraries and everything in-between for realising Algebraic Programming

Switzerland https://algebraic-programming.gith...

The ALPs:

- linear algebra: **ALP/GraphBLAS** and **ALP/Dense**,
- vertex-centric programming: **ALP/Pregel**,
- towards tensor algebra: **ALP/Tensors** and **ALP/Ascend**,
- ...

Interoperability with existing software:

- **ALP/Spark**;
- **ALP/Solver**, **ALP/SparseBLAS**, **ALP/SpBLAS**.

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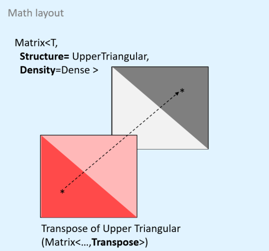

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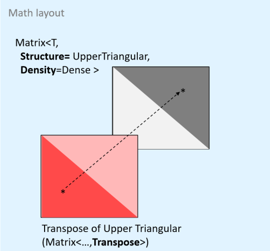

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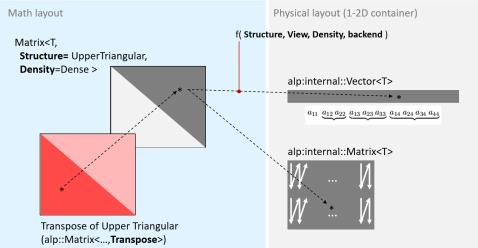

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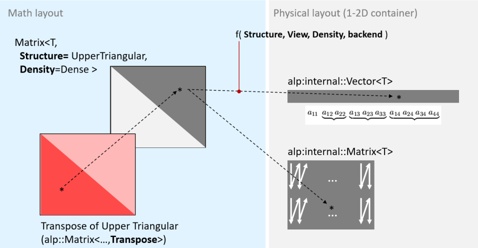

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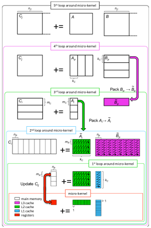

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Between JIT and AOT: **delayed compilation or dispatching**

- high-level MLIR as an architecture-agnostic representation
- can be generated at run-time, following dynamic user control flow.

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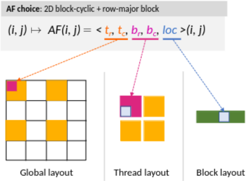
ALP/Dense implementation

ALP/Dense employs **block-wise storage** via composable xMFs:




```
#pragma omp parallel
{
  thread = config::OMP::current_thread_ID();
  th_coo = da.getThreadCoords( thread );
  blk_grd = da.getLocalBlockGridDims( th_coo );

  for( auto &blk_coo: blk_grd ) {
    // Assuming shmem::seq == reference, Aloc has
   type
    // Matrix< ... , reference >
    auto Aloc = internal::get_view< shmem::seq >(
        A, th_coo, blk_coo
      );
      // Calls func on L.21
      eWiseMul( Aloc, alpha, beta, Sr );
  } }
```

Fig.: storage illustration (left) and code sketch for $A \otimes \alpha \oplus \beta$ (right)

ALP/Dense:

- performs **compile-time simplification** of telescopic composed xMFs
- dispatches block computations to **sequential** hand-tuned BLAS
- performs **NUMA-aware allocation** of thread-owned blocks
- **blocking execution** within threads

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ALP/Dense evaluation

Expressivity: the API captures the following LA algorithms

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- ALP/Dense (block size 128) compared to Netlib LAPACK;
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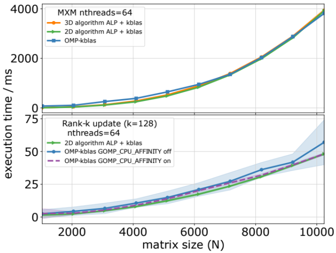

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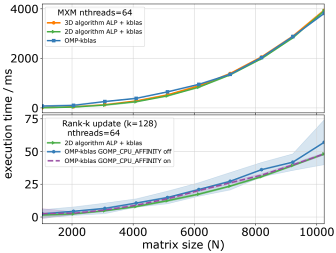

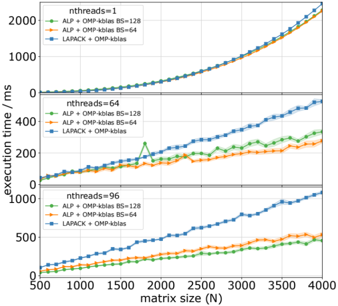

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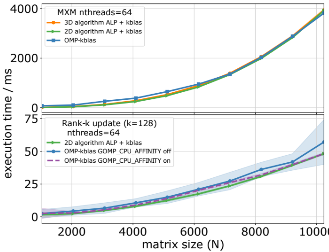
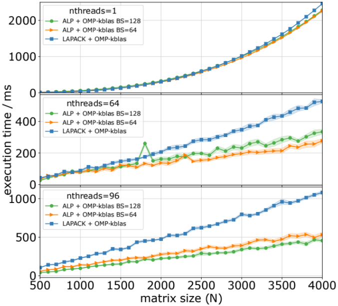
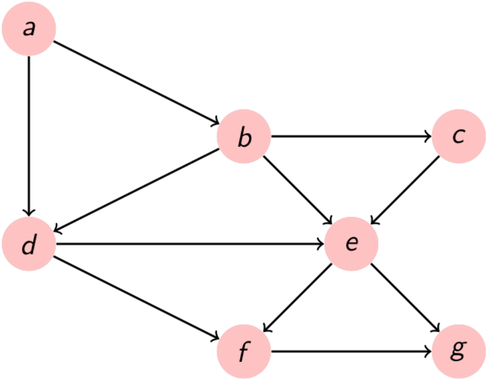




- ALP/Dense NUMA-aware auto-parallelisation: more stable, faster

ALP/Pregel

Pregel:



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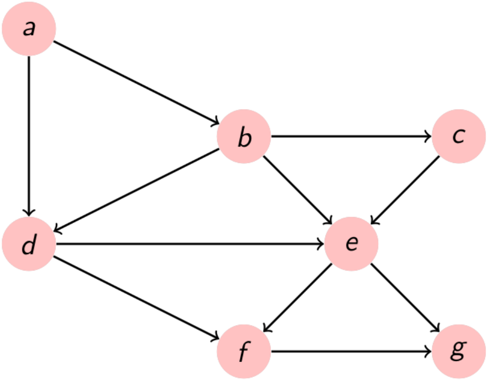

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Think like a vertex, Malewicz et al. '10.

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  - use local convergence detection and vote-to-halt.
- while "PageRank-like", **not mathematically equivalent**!

ALP/Pregel

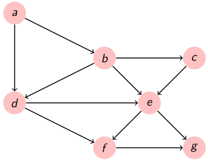
Expanding Pregel programs into ALP/GraphBLAS:

```
static void program(
        VertexIDType &current_max_ID,    // each vertex starts with its unique ID
  const VertexIDType &incoming_message,  // IDs will propagate from neighbours
        VertexIDType &outgoing_message,  // new max IDs will be broadcast
  grb::interfaces::PregelData &pregel
) {

  if( pregel.round > 0 ) {                     // messages arrive after round 1

    if( current_max_ID < incoming_message ) { // a larger ID has arrived; join the
      current_max_ID = incoming_message;       // component 'led' by this ID

    } else {                                   // otherwise no change: if everyone
      pregel.voteToHalt = true;                // has no change, stop execution
    }
  }
  outgoing_message = current_max_ID;           // as long as we're running, keep
}                                              // broadcasting my component ID
```



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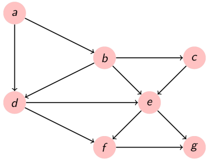

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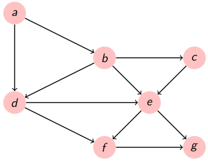

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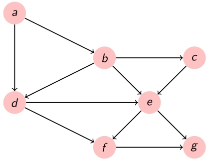

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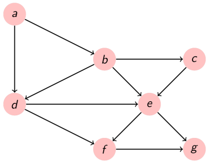

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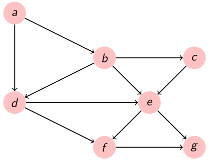

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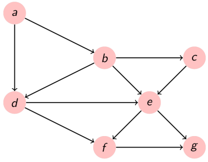

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grbcxx -b hybrid myPregelAlgo pregelAlgo.cpp
grbrun -b hybrid -np 4 ./myPregelAlgo
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For the Pregel "page-ranking", **two variants**:

- global: terminate when all vertices are converged;
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	ALP/Pregel		Sequential
Dataset	Global	Local	GraphBLAS
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G2_circuit	175 (38)	78.8 (36)	90.0 (48)
bundle_adj	3 070 (66)	2 070 (51)	2 330 (60)
G3_circuit	1 960 (38)	987 (36)	*1 100* (48)
wiki-2007	40 500 (103)	11 400 (96)	18 100 (55)
uk-2002	153 000 (115)	*46 100* (104)	*72 100* (73)
road_usa	*87 600* (78)	58 800 (72)	62 200 (78)

Sequential performance in ms. Compares different page ranking algorithms.

ALP/Pregel

When using the (blocking) shared-memory parallel backend:

Dataset	ALP/Pregel Global	ALP/Pregel Local	Blocking GraphBLAS
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G2_circuit	58.8 (38)	38.9 (36)	29.0 (48)
bundle_adj	280 (66)	*224* (51)	1 290 (60)
G3_circuit	367 (38)	243 (36)	87.8 (48)
wiki-2007	2 440 (103)	878 (96)	5 030 (55)
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- 0.84–13.0× speedup for the local Pregel page ranking;
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