

Algebraic Programming for High Performance Auto-Parallelised Solvers

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Humble and Hero Programming

- **Hero** programmers: achieve maximum efficiency;
- **Humble** programmers: achieve maximum productivity.

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Increasingly many hardware targets,
increasingly heterogeneous hardware:

a software productivity crisis is looming

Historical context

- The Design and Analysis of Computer Algorithms, Aho, Hopcroft, Ullman (1974)
- Introduction to Algorithms (first edition only), Cormen, Leiserson, Rivest (1990)
- **Elements of Programming**, Alexander Stepanov & Paul McJones (2009)
- **Graph Algorithms in the Language of Linear Algebra**, Jeremy Kepner & John Gilbert (2011)
- **From Mathematics to Generic Programming**, Alexander Stepanov & Daniel Rose (2015)
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- A C++ GraphBLAS, Y., Suijlen, Di Nardo, Nash (2017)
- **Algebraic Programming** (2021 onwards)
- ...

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- explicitly annotate computations with algebraic information;
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- automatically optimise code based on algebraic information;
- allow only scalable expressions.

Basics

Three ALP concepts: algebraic *containers*, *structures*, and *primitives*.

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```
grb::Vector< double > x( n ), y( n ), z( n );  
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```
grb::Vector< std::pair< int, double > > pairs( n );
grb::Vector< bool > s( n, 1 );           // nz cap: one
grb::Matrix< void > L( n, n, nz );       // nz cap: nz
```

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Elements may be **any POD type**, containers have **capacities** and **IDs**:

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grb::Vector< std::pair< int, double > > pairs( n );
grb::Vector< bool > s( n, 1 );           // nz cap: one
grb::Matrix< void > L( n, n, nz );       // nz cap: nz
std::cout << "s_has_ID" << grb::getID( s ) << "\n";
```

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Algebraic structures are **types**. E.g., $\text{min} : D_1 \times D_2 \rightarrow D_3$ reads

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> addMon;
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```

```
grb :: Semiring<
  grb :: operators :: add< double >,
  grb :: operators :: mul< double >,
  grb :: identities :: zero , grb :: identities :: one
> mySemiring;
```

Basics

Algebraic primitives operate on algebraic containers:

- `grb :: set( x, 1.0 );` `//  $x_i = 1, \forall i$`
- `grb :: setElement( y, 3.0, n/2 );` `//  $y_{n/2} = 3$`

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Semantics may change based on **required** algebraic structures:

- `grb :: eWiseApply( z, x, x, minOp );` `//  $z_i = \min\{x_i, x_i\}, \forall i$`
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All primitives except '**getters**' such as `grb :: { size , n rows, nnz, capacity }`:

- allow input & output **masks**, **descriptors**, and **phase** arguments;
- return **error codes** such as for mismatching dimensions.

Example: PageRank

Inner PageRank loop in ALP, following a textbook definition:

```

beta = gamma = 0;
foldl< invert_mask >( gamma, pr, r, add );
eWiseApply( u, pr, r, mul );
gamma = ( alpha * gamma + 1 - alpha ) / n;
set( t, 0 ); vxm( t, u, L, plusTimes );
foldl( t, gamma, add );
dot( beta, pr, t, add, absDiff );
std::swap( pr, t ); ++iter;
if( beta <= tol || iter == max_iter ) { break; }

```

```

// gamma = pr( !r ) * e^T
// u = pr .* r
// standard scalar arith.
// t = u * L
// t = t .+ gamma
// beta = || pr - t ||_1

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Selecting the **sequential auto-vectorising** backend:

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grbcxx -o myProgram myProgram.cpp  
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Selecting the **shared-memory parallel auto-vectorising** backend:

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grbcxx --backend reference_omp -o myProgram myProgram.cpp  
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Selecting the **1D distributed-memory parallel** backend (4 nodes):

```
grbcxx -b bsp1d -o myProgram myProgram.cpp
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```
grbrun -b bsp1d -np 4 ./myProgram datasets/west0497.mtx
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Selecting the **hybrid shared and dist. parallel** backend (10 nodes):

```
grbcxx -b hybrid -o myProgram myProgram.cpp
```

```
grbrun -b hybrid -np 10 ./myProgram datasets/west0497.mtx
```

The nonblocking backend

Suppose we compute $s = r + \alpha v$ over a given semiring:

- 1) `grb :: set( s, r );` `// s = r`
- 2) `grb :: eWiseMul( s, alpha, v, semiring );` `// s += alpha .* v`

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Manual fusion (Y. et al., '20): performance ✓

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grb::eWiseLambda( [ &s, &r, &alpha, &v, &ring ] (const size_t i) {
    grb::apply( s[ i ], alpha, v[ i ], ring.getMultiplicativeOperator() );
    grb::foldl( s[ i ], r[ i ], ring.getAdditiveOperator() );
}, s, r, v );
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Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by Y., D. Di Nardo, J. M. Nash, and W. J. Suijlen (2020).

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Automatic non-blocking mode (Mastoras et al., '22):

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Suppose we compute $s = r + \alpha v$ over a given semiring:

- 1) `grb::set( s, r );` `// s = r`
- 2) `grb::eWiseMul( s, alpha, v, semiring );` `// s += alpha .* v`

Blocking execution: the vector s is accessed *twice*; performance ✗

Manual fusion (Y. et al., '20): performance ✓, not very humble ✗

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grb::eWiseLambda( [ &s, &r, &alpha, &v, &ring ] (const size_t i) {
    grb::apply( s[ i ], alpha, v[ i ], ring.getMultiplicativeOperator() );
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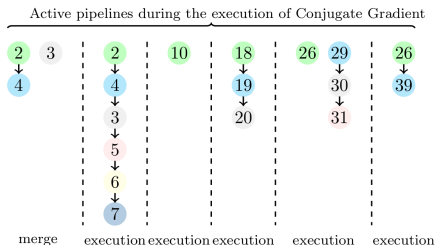
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- dynamically trigger pipelines when required, **automatically fuse**.

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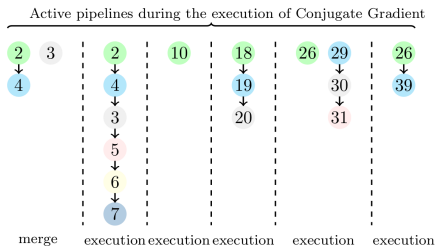
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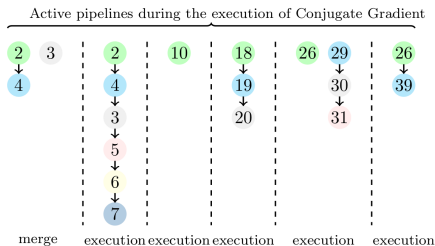
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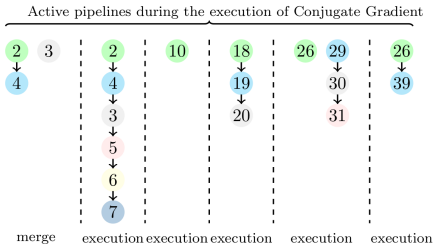
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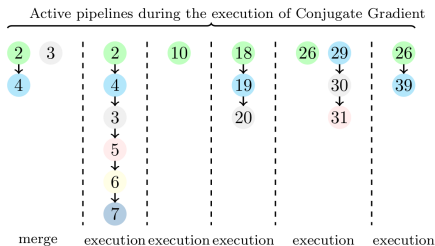
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Computing Systems Laboratory

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Similar results for PageRank and sparse deep neural network inference.

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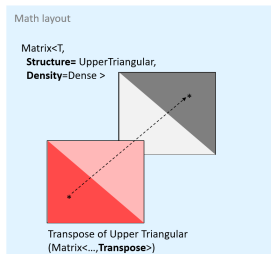
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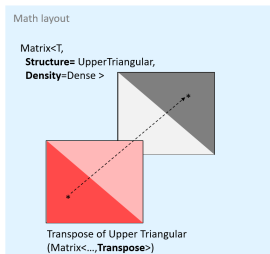
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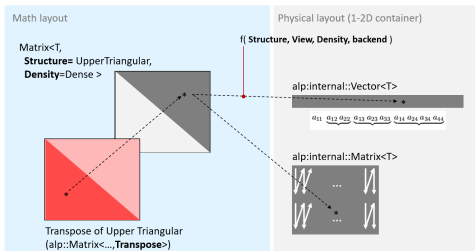
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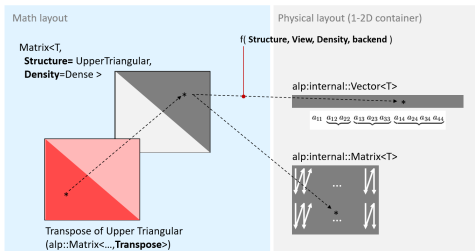
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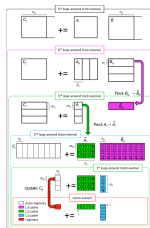
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ALP/Dense for solvers

Results:

- permutations, views, structures also **useful for sparse**

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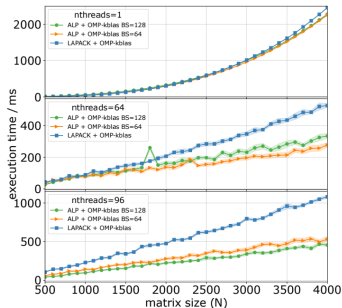
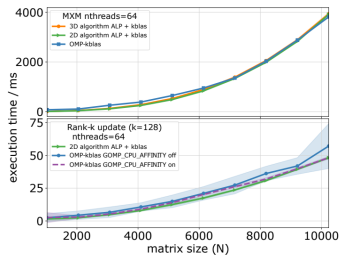
- permutations, views, structures also **useful for sparse**
- delayed compilation for dense LA: hard *and* unnecessary?

ALP/Dense for solvers

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Dense matrix–matrix multiplication (left)
and Cholesky decomposition (right)

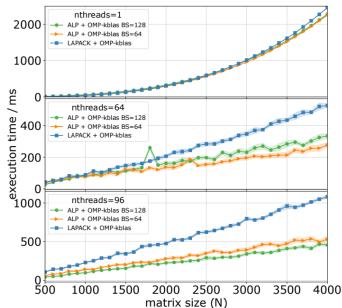
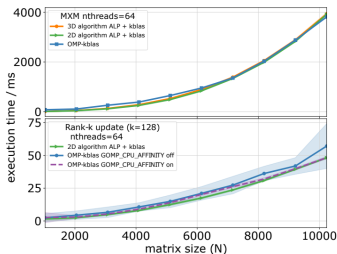


ALP/Dense for solvers

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Dense matrix–matrix multiplication (left)
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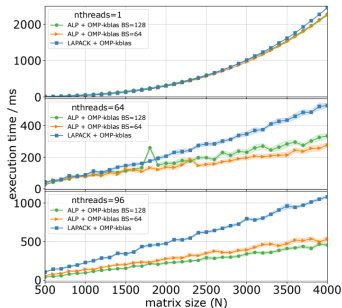
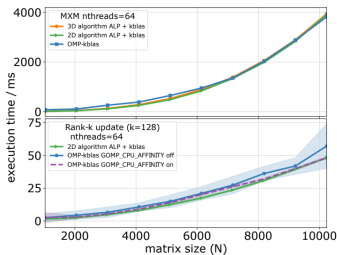
obtained via **dispatching to optimised BLAS(!)** Ref.: Spampinato et alii, '23

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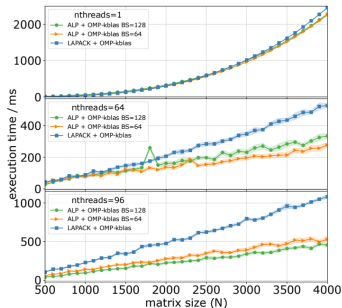
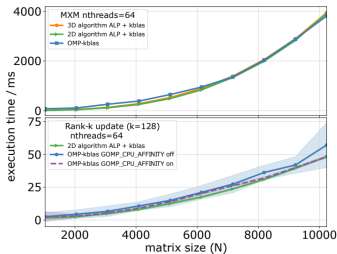
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Dense matrix–matrix multiplication (left)
and Cholesky decomposition (right)



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- **nonblocking** ALP/Dense expected to enhance performance;
- for sparse LA, hard-coded fusion in CG usually only **minor impact**.

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    grb::Vector< IOType > &buffer3, grb::Vector< IOType > &buffer4
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- CRS-compatible sparse iterative solvers:

```
double *x, *b, *a_vals; int *a_cols, *a_offs;
// ...

sparse_cg_handle_t handle;
sparse_cg_init( &handle, n, a_vals, a_cols, a_offs );

sparse_cg_solve( handle, x, b );
sparse_cg_destroy( handle );
```

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- CRS-compatible solvers with **user-defined preconditioning**:

```
double *x, *b, *a_vals; int *a_cols, *a_offs;
int my_preconditioner( double * out, const double * in, void * data );
// ...
sparse_cg_handle_t handle;
sparse_cg_init( &handle, n, a_vals, a_cols, a_offs );
sparse_cg_set_preconditioner( handle, my_preconditioner, data );
sparse_cg_solve( handle, x, b );
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```

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ALP generates the following libs:

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- alp_cspblas and alp_cspblas_shmem_parallel
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Eigen	15.1	10.5	22.3	20.0	28.1	99.2
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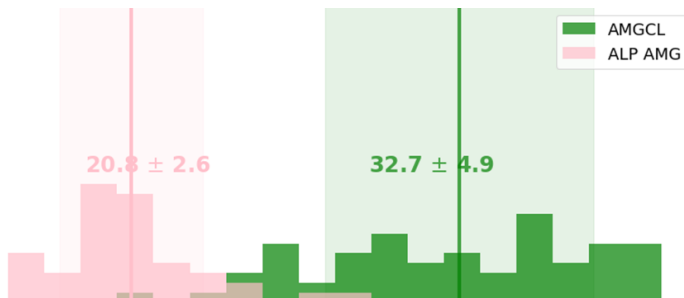
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ALP up to **28.5×** faster vs. Eigen and **6.96**, **2.01×** vs. hand-optimised.

ALP PCG with AMG preconditioning

AMG PCG in ALP vs. AMGCL on an internal problem:

- Matrix size n approx. 217M, 460M nonzeros.
- explicit coarsening / restriction matrices applied within ALP;
- compiled using the non-blocking backend.



(obtained using the 2-socket ARM machine, 96 cores; similar for x86)

ALP PCG with ILU(tresh)

Tresholded ILU PCG in ALP vs. Eigen on the same internal problem:

- forward/backward solves $Lx = b$ and $L^T y = x$ are outside of ALP;
 - optimised using Sympiler/HDAGG
- they are called from the standard ALP PCG algorithm.

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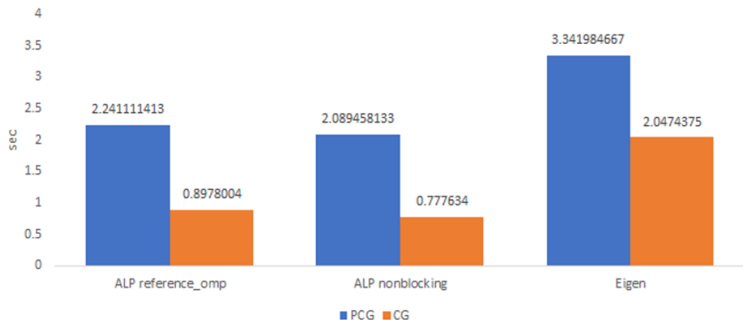


(obtained on a 2-socket Intel x86 machine, 32 threads)

ILU(tresh) PCG on x86

Two-socket Intel x86, smaller problem

- Matrix size n approx. 44M, 100M nonzeros.

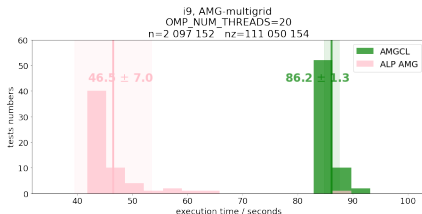
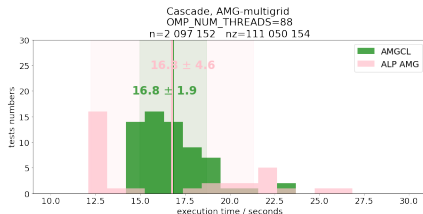


(obtained on a 2-socket Intel x86 machine, 32 threads)

Here, PCG Eigen: 3.36, Eigen+Hdag: 2.08, ALP+Hdag: 0.778 s.

AMG PCG on x86

Two-socket and one-socket performance on Intel x86:



- Cascade Lake, 64-byte vectorisation, default ALP settings

HPCG: multi-node benchmark mimicking AMG

HPCG benchmark, dual-socket ARM, 96 cores, maximum problem size

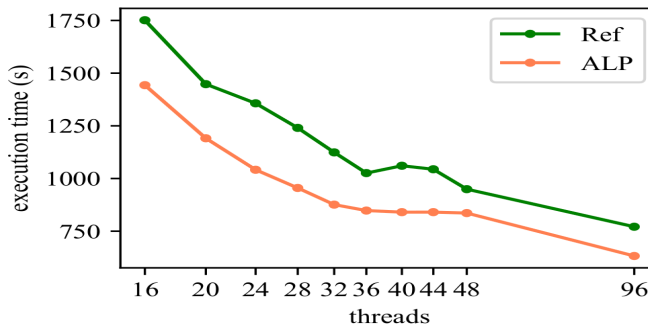
- **reference HPCG code** modified to use **Red-Black Gauss-Seidel**,
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HPCG benchmark, dual-socket ARM, 96 cores, maximum problem size

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Comparison, using the blocking ALP backend:



Ref. Scolari, Y.: "Effective implementation of the High Performance Conjugate Gradient benchmark on ALP/GraphBLAS", GrAPL at IPDPSW (to appear, 2023)

ALP as a foundational programming model



The ALPs:

- ALP/GraphBLAS, ALP/Pregel, ALP/Dense, ...

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Integrates with existing software:

- interoperability: ALP/Spark;
- transition paths: ALP/SparseBLAS, ALP/SpBLAS, ALP/Solver.

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Future:

- what other humble APIs can ALP efficiently simulate?
- what other algebras to support for widened applicability?

ALP as a foundational programming model



The ALPs:

- ALP/GraphBLAS, ALP/Pregel, ALP/Dense, ...

Integrates with existing software:

- interoperability: ALP/Spark;
- transition paths: ALP/SparseBLAS, ALP/SpBLAS, ALP/Solver.

Future:

- what other humble APIs can ALP efficiently simulate?
- what other algebras to support for widened applicability?

One software stack, many humble interfaces, hero performance



Ref.: Y., "Humble Heroes", Communications of Huawei Research.

Computing Systems Laboratory

A. N. Yzelman

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It's open!

Open source, Apache 2.0, welcome to try, use, and collaborate!

- <https://github.com/Algebraic-Programming>
- <https://algebraic-programming.github.io>



Publications:

- Suijlen, Y.: Lightweight Parallel Foundations: a model-compliant communication layer (2019);
- Y., Di Nardo, Nash, Suijlen: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation (2020);
- Mastoras, Anagnostidis, Y.: Nonblocking execution in GraphBLAS, IPDPSW (2022);
- Chelini, Barthels, Bientinesi, Copic, Grosser, Spampinato: MOM: Matrix Operations in MLIR, HiPEAC IMPACT workshop (2022);
- Y.: Humble Heroes, Communications of Huawei Research (2023, to appear);
- Mastoras, Anagnostidis, Y.: Design and implementation for nonblocking execution in GraphBLAS: tradeoffs and performance, ACM TACO (2023);
- Scolari, Y.: Effective implementation of the High Performance Conjugate Gradient benchmark on ALP/GraphBLAS, IPDPSW (GrAPL 2023);
- Spampinato, Jelovina, Zhuang, Y.: Towards Structured Algebraic Programming, ACM ARRAY (2023);
- Papp, Anegg, Y.: Partitioning Hypergraphs is Hard: Models, Inapproximability, and Applications, ACM SPAA (2023);
- Papp, Anegg, Y.: DAG scheduling in the BSP model (submitted, 2023);
- Pasadakis, et al., Nonlinear spectral clustering with C++ GraphBLAS, extended abstract, IEEE HPEC (2023, outstanding short paper);
- Papp, Anegg, Karanasiou, Y.: Efficient Multi-Processor Scheduling in Increasingly Realistic Models (submitted, 2023).

Backup slides

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Performance

Scale-out performance of graph algorithms, using the hybrid backend:

- Clueweb12 link matrix, approx. 978M vertices and 42.5B edges

	Ivy Bridge nodes						
	4	5	6	7	8	9	10
Input	1524	1271	1067	943	691	662	537
4-hop reachability BFS	48.8	110	54.8	99.6	83.0	74.2	23.3
20-hop reachability BFS	404	280	231	323	221	230	160
PageRank	13.3	10.3	9.68	8.00	21.0	22.9	21.6

The k -hop BFS and PageRank (PR) on Clueweb12, performance in seconds. Infiniband EDR interconnect.

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Transition path: CG, x86, ALP v0.5

ms. (and speedup) per CG iteration on x86 (2 CPUs, 88 hyperthreads):

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- **guide programmers** to express the best possible algorithm;
- **gauge scalability**: compute resources vs. problem size;
- expose **trade-off opportunities**: e.g., speed vs. memory;
- **automatic choice** of algorithms and backends.

Performance semantics

Every ALP program can be **systematically costed**:

Primitive	Work	Ops	Data movement	Reductions
<code>setElement(x, y, i)</code>	1	-	1	no
<code>set(x, y)</code>	$\min\{n, nz_x + nz_y\}$	-	$nz_x + nz_y$ or $n + nz_y$	no
<code>clear(x)</code>	nz_x	-	nz_x	no
<code>apply(z, x, y, $\odot/M$)</code>	$\min\{n, nz_x + nz_y\}$	$nz_x \cap y$	$2 \min\{n, nz_x + nz_y\} + nz_{x \cup y}$	no
<code>foldl(y, x, $\odot/M$)</code> <code>foldr(x, y, $\odot/M$)</code>	nz_x	$nz_x \cap y$	$2nz_x$	no
<code>foldl(y, α, $\odot/M$)</code> <code>foldr(α, y, $\odot/M$)</code> <code>foldl(α, y, M)</code> <code>foldr(y, α, M)</code>	nz_y	nz_y	nz_y	no yes
<code>mul(z, x, y, R)</code>	$\min\{nz_x, nz_y\}$	$nz_x \cap y$	$2 \min\{nz_x, nz_y\} + nz_{x \cap y}$	no
<code>dot(z, x, y, (M, $\odot$))</code> <code>dot(z, x, y, R)</code>	n $\min\{nz_x, nz_y\}$	$2n$ $2 \cdot nz_x \cap y$	$2n$ $2 \min\{nz_x, nz_y\}$	yes

Level-1 primitives and their costs, excluding masking. Similar tables exist for level-2 and level-3 primitives.

Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by Y., D. Di Nardo, J. M. Nash, and W. J. Suijlen (2020).

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Every container has **memory use semantics**:

- “static” costs proportional to container sizes;
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Capacities are **optional** during container construction:

```
grb :: Vector< bool > s( n, 1 );  
grb :: Matrix< void > L( n, n, nz );
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Capacities:

- are **lower bounds**; $\text{grb} :: \text{capacity}(s) \geq 1$;
- may **increase** through `grb :: resize`, updates memory use semantics;
- Any request to decrease capacity thus **may be ignored**.

Algebraic type traits

Algebraic type traits: compile-time introspection of algebraic info

- `grb :: is_associative < Operator >::value`, `true` iff $(a \odot b) \odot c = a \odot (b \odot c)$;
- `grb :: is_idempotent < Operator >::value`, `true` iff $a \odot a = a$;
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These are all **compile-time constant** (through C++11 `constexpr`):

- similar to the standard C++11 *type traits*.

Algebraic type traits

Algebraic type traits help

- detect programmer errors,
- decide which optimisations are applicable, and
- reject expressions without recipe for auto-parallelisation.

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For example, **algebraic type traits prevent**

- creating a monoid from non-associative operator;
 - `Monoid< operators::divide<int>, identities :: one > myMonoid;`
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These optimisations are applied **at compile-time**,
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Ex.: Y & Bisseling '10; Y & Roose '14; Y, Bisseling, Roose, Meerbergen '14; Y, Roose, Meerbergen '14; ...