Algebraic Programming

Portable, high-performance, and high-productivity programming

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Humble and Hero Programming

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a software productivity crisis is looming

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- **re-define compiler, library, and application boundaries**!

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- automatically **optimise** code based on algebraic information;
- allow only **scalable** expressions.

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Three ALP concepts: algebraic *containers*, *structures*, and *primitives*.

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Containers are similar to the standard template library (STL):

```
grb::Vector< double > x( n ), y( n ), z( n );
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Elements may be **any POD type**, containers have **capacities** and **IDs**:

```
grb::Vector< std::pair< int, double > > pairs( n );
grb::Vector< bool > s( n, 1 );        // nz cap: one
grb::Matrix< void > L( n, n, nz ); // nz cap: nz
std::cout << "s has ID " << grb::getID( s ) << "\n";
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grb::Semiring<
    grb::operators::add< double >,
    grb::operators::mul< double >,
    grb::identities::zero,
    grb::identities::one
> mySemiring;
```

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Algebraic primitives operate on algebraic containers:

- grb :: set(x, 1.0); $\quad // \; x_i = 1, \; \forall i$
- grb :: setElement(y, 3.0, n/2); $\quad // \; y_{n/2} = 3$

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Semantics may change based on **required** algebraic structures:

- grb :: eWiseApply(z, x, y, minOp); // $z_i = \min\{x_i, y_i\}$, for $i = n/2$
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- grb::mxv(y, A, x, mySemiring); // y += Ax; in-place

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All primitives except '**getters**' such as grb :: { size , nrows, nnz, capacity }:

- allow output **masks**, **descriptors**, and **phase** arguments;
- return **error codes** such as for mismatching dimensions.

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Algebraic type traits: compile-time introspection of algebraic info

- grb :: is_associative< Operator >::value, `true` iff $(a \odot b) \odot c = a \odot (b \odot c)$;
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  therefore

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These are all **compile-time constant** (through C++11 constexpr):

- similar to the standard C++11 *type traits*.

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Algebraic type traits help

- detect programmer errors,
- decide which optimisations are applicable, and
- reject expressions without recipe for auto-parallelisation.

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- reducing a sparse vector to a scalar without a monoid structure.
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These optimisations are applied at compile-time,

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- replace primitives with cheaper ones:
  - grb::eWiseApply(z, x, x, S); sets $z_i = x_i \odot x_i$;
  - $\odot = \min$? Replace eWiseApply with grb::set(z, x);!
  - every time S is **idempotent**;
- ...

These optimisations are applied at compile-time,

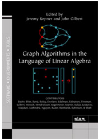
without requiring programmer knowledge or intervention.

Ex.: Y & Bisseling '10; Y & Roose '14; Y, Bisseling, Roose, Meerbergen '14; Y, Roose, Meerbergen '14; ...

Historical context

- The Design and Analysis of Computer Algorithms, Aho, Hopcroft, Ullman (1974)
- Introduction to Algorithms (first edition only), Cormen, Leiserson, Rivest (1990)
- **Elements of Programming**, Alexander Stepanov & Paul McJones (2009)
- Graph Algorithms in the Language of Linear Algebra, Jeremy Kepner & John Gilbert (2011)
- From Mathematics to Generic Programming, Alexander Stepanov & Daniel Rose (2015)
- **GraphBLAS.org**, following work by Kepner & Gilbert, Kepner, Gilbert, Buluç, Mattson, et alii (2016)
- A C++ GraphBLAS, Y. et al. (2017–2020)
- **Algebraic Programming** (2021 onwards)
- ...

GraphBLAS



`GraphBLAS.org`; Kepner, Gilbert, Buluç, Mattson, Moreira, ...

- for example, $y = A^k x$, parametrised in a semiring:

```
template< typename Semiring, typename NonzeroT, typename VectorT >
grb::RC mpv(
              grb::Vector< VectorT >   &y,
        const grb::Matrix< NonzeroT > &A, const size_t k,
        const grb::Vector< VectorT >   &x,
              grb::Vector< VectorT >   &buffer,
        const Semiring                 &ring = Semiring()
) {
        // error checking and error propagation omitted
        grb::vxm( y, x, A, ring );
        for( size_t i = 1; i < k; ++i ) {
                std::swap( y, buffer );
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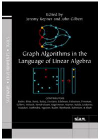
GraphBLAS



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Solve different problems with different semirings:

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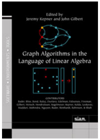

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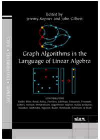

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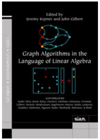

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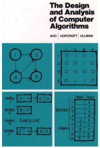

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Single-source shortest-paths (SSSP) using a min-plus semiring:

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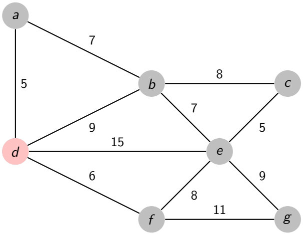
$$\underbrace{\begin{pmatrix} \\ \\ \\ 0 \\ \\ \\ \\ \end{pmatrix}}_{x+=Ax} += \underbrace{\begin{pmatrix} x & 7 & & 5 & & & \\ 7 & x & 8 & 9 & 7 & & \\ & 8 & x & & 5 & & \\ 5 & 9 & & x & 15 & 6 & \\ & 7 & 5 & 15 & x & 8 & 9 \\ & & & 6 & 8 & x & 11 \\ & & & & 9 & 11 & x \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \\ \\ \\ 0 \\ \\ \\ \\ \end{pmatrix}}_{x}$$



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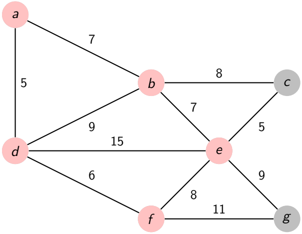

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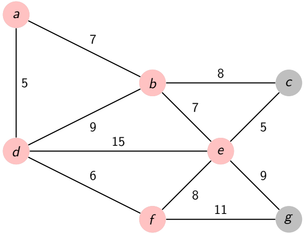

Ax: shortest distances within *one* hop:

- x contains one nonzero, select corresponding adjacent vertices;
- compute sum with inputs, reduce into output using min.

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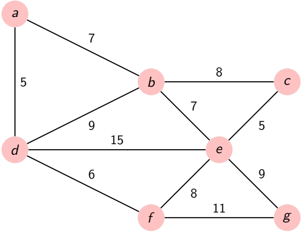

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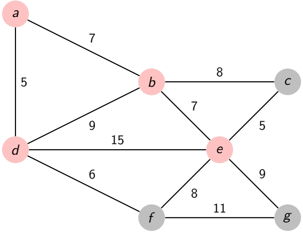

$A^2 x$: shortest distances within *two* hops:

- **swap** input and output vectors, repeat procedure;
- multiple paths reach e.g. b: $\min\{5 + 7 = 12, 9, 15 + 7 = 22\} = 9$.

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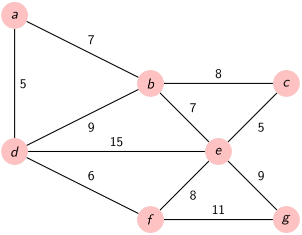
$$\underbrace{\begin{pmatrix} 5 \\ \mathbf{9} \\ 17 \\ {\color{blue}0} \\ 14 \\ 6 \\ 17 \end{pmatrix}}_{Ax += A^3x} += \underbrace{\begin{pmatrix} x & 7 & & 5 & & & \\ 7 & x & 8 & 9 & 7 & & \\ & 8 & x & & 5 & & \\ & 9 & & x & 15 & 6 & \\ & 7 & 5 & 15 & x & 8 & 9 \\ & & & 6 & 8 & x & 11 \\ & & & & 9 & 11 & x \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 5 \\ 9 \\ 17 \\ 0 \\ 14 \\ 6 \\ 17 \end{pmatrix}}_{A^2x}$$



A^3x: shortest distances within three hops:

- **swap** input and output vectors, repeat procedure;
- output equals input vector; SSSP solved, **terminate**.

GraphBLAS

A non-exclusive list of graph algorithms expressed using GraphBLAS:

- strongly connected components,
- maximal independent set,
- betweenness centrality,
- *k*-core decomposition,
- graph contraction,
- depth-first search,
- triangle counting,
- graph generation,
- graph clustering,
- shortest paths,
- ...and more– see `graphblas.org` for an up-to-date overview.

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1) specific backend for specific architectures or systems;

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Selecting the **sequential** **auto-vectorising** backend:

```
grbcxx -o myProgram myProgram.cpp
grbrun ./myProgram datasets/west0497.mtx
```

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Selecting the **shared-memory parallel** **auto-vectorising** backend:

```
grbcxx --backend reference_omp -o myProgram myProgram.cpp
grbrun -b reference_omp ./myProgram datasets/west0497.mtx
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Selecting the **1D distributed-memory parallel** backend (4 nodes):

```
grbcxx -b bsp1d -o myProgram myProgram.cpp
grbrun -b bsp1d -np 4 ./myProgram datasets/west0497.mtx
```

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Selecting the **hybrid shared and dist. parallel** backend (10 nodes):

```
grbcxx -b hybrid -o myProgram myProgram.cpp
grbrun -b hybrid -np 10 ./myProgram datasets/west0497.mtx
```

Performance semantics

Every ALP backend defines **performance semantics**:

- work;

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- guide programmers to express the best possible algorithm;
- gauge scalability: compute resources vs. problem size;
- expose trade-off opportunities: e.g., speed vs. memory;
- automatic choice of algorithms and backends.

Performance semantics

Every ALP program can be systematically costed:

Primitive	Work	Ops	Data movement	Reductions
setElement(x, y, i)	1	-	1	no
set(x, y)	$\min\{n, nz_x + nz_y\}$	-	$nz_x + nz_y$ or $n + nz_y$	no
clear(x)	nz_x	-	nz_x	no
apply$(z, x, y, \odot/M)$	$\min\{n, nz_x + nz_y\}$	$nz_{x\cap y}$	$2\min\{n, nz_x + nz_y\} + nz_{x\cup y}$	no
foldl$(y, x, \odot/M)$	nz_x	$nz_{x\cap y}$	$2nz_x$	no
foldr$(x, y, \odot/M)$	nz_x	$nz_{x\cap y}$	$2nz_x$	no
foldl$(y, \alpha, \odot/M)$	nz_y	nz_y	nz_y	no
foldr$(\alpha, y, \odot/M)$	nz_y	nz_y	nz_y	no
foldl(α, y, M)	nz_y	nz_y	nz_y	yes
foldr(y, α, M)	nz_y	nz_y	nz_y	yes
mul(z, x, y, R)	$\min\{nz_x, nz_y\}$	$nz_{x\cap y}$	$2\min\{nz_x, nz_y\} + nz_{x\cap y}$	no
dot$(z, x, y, (M, \odot))$	n	$2n$	$2n$	yes
dot(z, x, y, R)	$\min\{nz_x, nz_y\}$	$2 \cdot nz_{x\cap y}$	$2\min\{nz_x, nz_y\}$	yes

Level-1 primitives and their costs, excluding masking. Similar tables exist for level-2 and level-3 primitives.

Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by Y., D. Di Nardo, J. M. Nash, and W. J. Suijlen (2020).

Sparse vectors

Sparse vectors:

- ideal: $\mathcal{O}(1)$ query, assign, and iteration.
- **sparse accumulators**: nonzero index **stack** and boolean **array**;
- in parallel: synchronise, combine sparsity structures. **Prefix-sum**.

Gilbert, Moler, and Schreiber, *Sparse matrices in MATLAB: Design and implementation.* SIAM JMAA (1992); Y et al., *A C++ GraphBLAS* (2020). Big-Oh bounds in the classical RAM model.

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Alternative, tree-based map (`std::map`):

- $\mathcal{O}(\log nz)$ query and assign;
- $\mathcal{O}(1)$ iteration.
- parallelisation: join, intersect (set algebra!)

Currently **not** implemented in ALP/GraphBLAS due to overhead.

Davis, *SuiteSparse::GraphBLAS: Graph algorithms in the language of sparse linear algebra*. ACM TOMS ('19).

Sparse matrices

Sparse matrices of size $m \times n$, nz nonzeroes:

- use $\Theta(nz)$ storage, not $\Theta(mn)$,
- level-2 cost $\sim$ number of nonzeroes touched,
- level-3 cost $\sim$ number of operator applications required.

Many sparse matrix storages exist.

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We use **Gustafson's format** (CRS+CCS).

Y. and Roose, *High-level strategies for sparse matrix–vector multiplication*, IEEE TPDS 2014.
Y et al., *A C++ GraphBLAS*, 2020.
Simecek, Langr, Tvrdik. *Minimal quadtree format for compression of sparse matrices storage.* SSNA (2012).

Containers

Data structures are **opaque**; data representations are chosen

fully automatically, hidden from user.

The nonblocking backend

Suppose we compute $s = r + \alpha v$ over a given `semiring`:

1) `grb::set( s, r );`
2) `grb::eWiseMul( s, alpha, v, semiring );`

Blocking execution: the vector s is accessed *twice*

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2) `grb::eWiseMul( s, alpha, v, semiring );`

Blocking execution: the vector s is accessed *twice*; performance ✗

Manual fusion (Y. et al., '20): performance ✓

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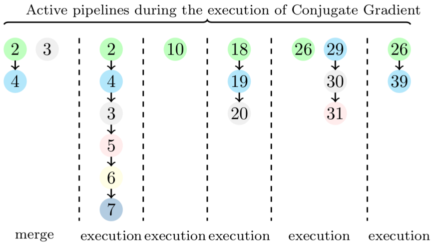


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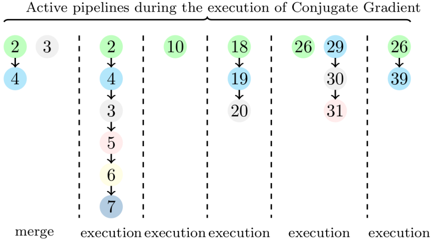

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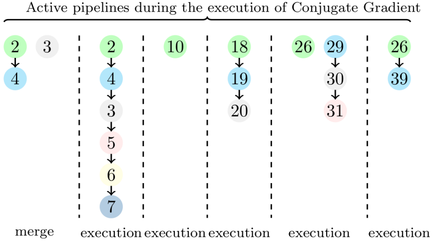


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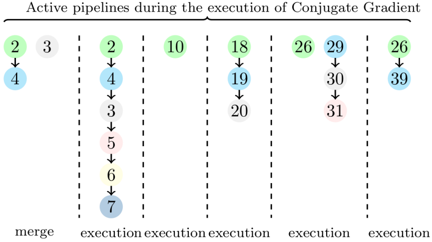

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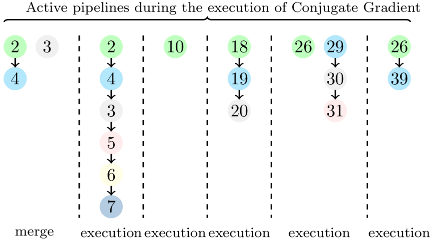


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Performance

Speedup relative to sequential ALP (v0.5), vs. state-of-the-art

- Conjugate Gradient solve, two-socket x86, 44 cores:

	gyro_m	G2_circuit	bundle_adj	ecology2	Queen_4147
GSL	0.84	0.95	0.89	0.91	0.92
blocking ALP	2.30	4.53	**12.7**	6.91	17.5
SuiteSparse:GraphBLAS	1.57	1.11	5.82	3.52	11.6
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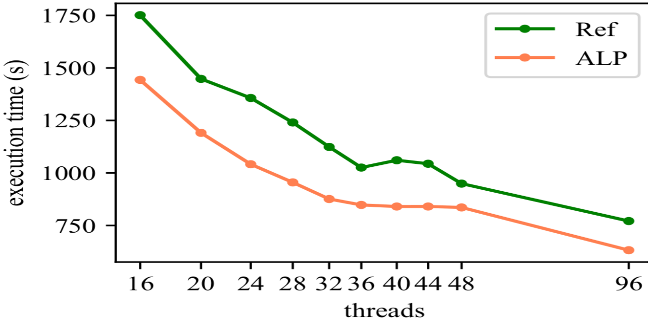

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Scale-out performance of graph algorithms, using the hybrid backend:

- Clueweb12 link matrix, approx. 978M vertices and 42.5B edges

	Ivy Bridge nodes						
	4	5	6	7	8	9	10
Input	1524	1271	1067	943	691	662	537
4-hop reachability BFS	48.8	110	54.8	99.6	83.0	74.2	23.3
20-hop reachability BFS	404	280	231	323	221	230	160
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Performance

Scale-out performance of graph algorithms, using the hybrid backend:

- Clueweb12 link matrix, approx. 978M vertices and 42.5B edges

	Ivy Bridge nodes						
	4	5	6	7	8	9	10
Input	1524	1271	1067	943	691	662	537
4-hop reachability BFS	48.8	110	54.8	99.6	83.0	74.2	23.3
20-hop reachability BFS	404	280	231	323	221	230	160
PageRank	13.3	10.3	9.68	8.00	21.0	22.9	21.6

The k-hop BFS and PageRank (PR) on Clueweb12, performance in seconds. Infiniband EDR interconnect.

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Orders of magnitude improvements on **10 nodes** (hybrid backend):

				Spark				Spark with ALP/GraphBLAS			
	GB	Gnz	n_ϵ	$n=1$	$n=10$	$n=n_\epsilon$	s/it.	$n=1$	$n=10$	$n=n_\epsilon$	s/it.
uk-2002	4.7	0.3	73	168.6	1373.8	>4 hrs	133.9	8.7	13.9	48.7	0.56
clueweb12	786	42.5	45	-	-	-	-	658.8	963.2	1875.0	27.7

Pagerank performance in seconds using ten Ivy nodes with Infiniband EDR, Spark 2.3.1, and Hadoop 2.7.7.

- I/O: **19×** faster, computation: **239×** faster for `uk-2002`;

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- I/O: **19× faster**, computation: **239× faster** for `uk-2002`;
- Spark Clueweb: out of memory; Blogel (Ammar, Ozsu '18): 128 nodes
  - can handle **141×** larger problems, **12×** fewer resources

Ref.: Suijlen and Y., "Lightweight Parallel Foundations: a model-compliant communication layer", (2019; pre-v0.1 ALP. Results are being refreshed with latest ALP, Scala, Spark, LPF: see ALP/Spark @ GitHub).

Beyond ALP/GraphBLAS

How far can we take this type of programming?

The Alps

The Alps:

- Monte Rosa,
- Matterhorn,
- Weisshorn,
- Jungfrau,
- Rothorn,
- Dom,
- ...

The ALPs

Algebraic Programming

IRs, communication layers, domain-specific languages, libraries and everything in-between for realising Algebraic Programming

Switzerland https://algebraic-programming.gith...

The ALPs:

- ALP/GraphBLAS,
- **ALP/Dense**,
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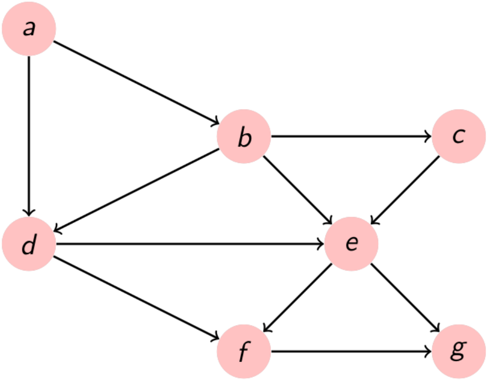
Interoperability with existing software:

- ALP/Spark;
- ALP/SparseBLAS, ALP/SpBLAS.

ALP/Pregel

Pregel:



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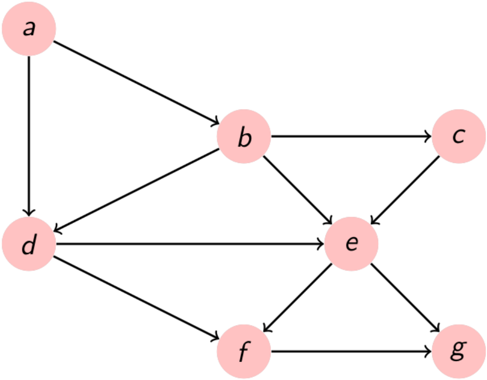

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Think like a vertex, Malewicz et al. '10.

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While “PageRank-like”, *not* mathematically equivalent!

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        VertexIDType &current_max_ID,     // each vertex starts with its unique ID
  const VertexIDType &incoming_message,   // IDs will propagate from neighbours
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  grb::interfaces::PregelData &pregel
) {

  if( pregel.round > 0 ) {                         // messages arrive after round 1

    if( current_max_ID < incoming_message ) {      // a larger ID has arrived; join the
      current_max_ID = incoming_message;           // component 'led' by this ID

    } else {                                       // otherwise no change: if everyone
      pregel.voteToHalt = true;                    // has no change, stop execution
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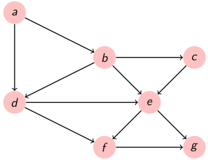

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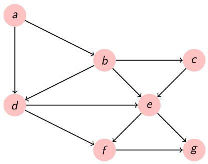

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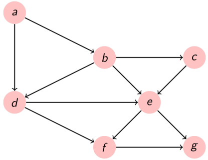

The translation is **automatic**, using the standard ALP software stack:

- `grbcxx -b hybrid myPregelAlgo pregelAlgo.cpp`
- `grbrun -b hybrid -np 4 ./myPregelAlgo`

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For the Pregel page ranking, **two variants**:

- terminate when all vertices are converged (global);
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Dataset	ALP/Pregel Global	ALP/Pregel Local	Sequential GraphBLAS
gyro_m	34.8 (40)	**24.7** (39)	31.4 (52)
G2_circuit	175 (38)	**78.8** (36)	90.0 (48)
bundle_adj	3 070 (66)	**2 070** (51)	2 330 (60)
G3_circuit	1 960 (38)	**987** (36)	*1 100* (48)
wiki-2007	40 500 (103)	**11 400** (96)	18 100 (55)
uk-2002	153 000 (115)	*46 100* (104)	*72 100* (73)
road_usa	*87 600* (78)	**58 800** (72)	62 200 (78)

Sequential performance in ms. Compares different page ranking algorithms.

ALP/Pregel

Same table, using the blocking shared-memory parallel backend:

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- 1.03–17.5× speedup for connected components algorithm.

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If generalised sparse linear algebra is so useful, **what about dense**?

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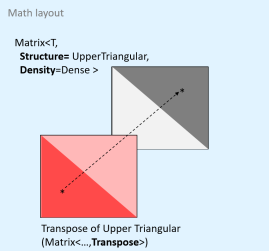

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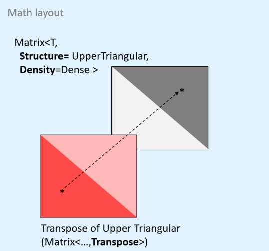

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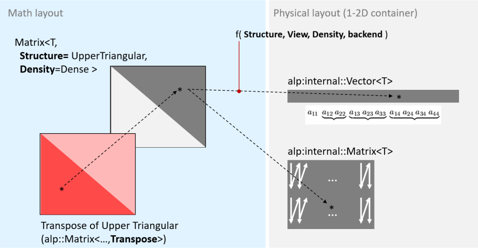

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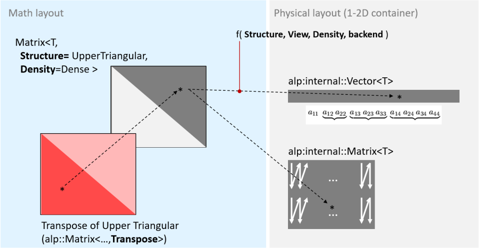
  - but also: outer products (two vectors), constant vectors (scalar)
- opaqueness: ALP controls layout, requires parametric **xMFs**



Ref.: Spampinato, Jelovina, Zhuang, Y: Towards Structured Algebraic Programming, ARRAY (2023)

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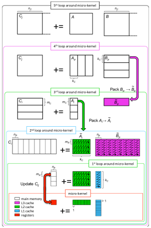

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One software stack with

multiple **humble interfaces**,

achieving **hero performance**.

It's open!

Open source, Apache 2.0, welcome to try, use, and collaborate!

- `https://github.com/Algebraic-Programming`
- `https://algebraic-programming.github.io`

Publications:

- Suijlen, Y.: Lightweight Parallel Foundations: a model-compliant communication layer (2019);
- Y., Di Nardo, Nash, Suijlen: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation (2020);
- Mastoras, Anagnostidis, Y.: Nonblocking execution in GraphBLAS, IPDPSW (2022);
- Chelini, Barthels, Bientinesi, Copic, Grosser, Spampinato: MOM: Matrix Operations in MLIR, HiPEAC IMPACT workshop (2022);
- Y.: Humble Heroes, Communications of Huawei Research (2023, to appear);
- Mastoras, Anagnostidis, Y.: Design and implementation for nonblocking execution in GraphBLAS: tradeoffs and performance, ACM TACO (2023);
- Scolari, Y.: Effective implementation of the High Performance Conjugate Gradient benchmark on ALP/GraphBLAS, GrAPL at IPDPSW (2023, to appear);
- Spampinato, Jelovina, Zhuang, Y.: Towards Structured Algebraic Programming, ACM ARRAY (2023);
- Papp, Anegg, Y.: Partitioning Hypergraphs is Hard: Models, Inapproximability, and Applications, ACM SPAA (2023);
- Papp, Anegg, Y.: DAG scheduling in the BSP model (preprint, 2023);
- Pasadakis, et al., Nonlinear spectral clustering with C++ GraphBLAS, extended abstract, IEEE HPEC (2023, **outstanding short paper**);
- Papp, Anegg, Karanasiou, Y.: Efficient Multi-Processor Scheduling in Increasingly Realistic Models (under preparation, 2023).

Backup slides

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Performance

Sparse Deep Neural Network inference, vs. sequential ALP

- GraphChallenge model & data

#layers	#neurons	GSL	Eigen	SS:GrB	non-blocking ALP
1920	64k	0.81×	7.91×	3.98×	**10.8**×
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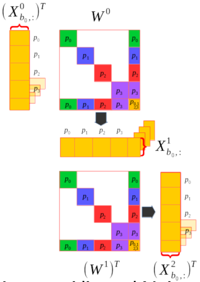
Jointly **partition** sparse layers, then tile across layers:

- work by Filip Pawłowski with Uçar and Bisseling
- 5 layers, 64k n.: **1.94**× speedup vs. data-parallel
- 2020 MIT/IEEE GraphChallenge innovation award
- combine with non-blocking ALP/GraphBLAS?



Ref.: Nonblocking execution in GraphBLAS by Aristeidis Mastoras, Sotiris Anagnostidis, and Y. in 2022 IEEE International Parallel and Distributed Processing Symposium Workshops (IPDPSW).
Ref.: Combinatorial Tiling for Sparse Neural Networks by F. Pawłowski, R. H. Bisseling, B. Uçar, Y. in 2020 IEEE High Performance Extreme Computing Conference (IEEE HPEC)

Performance semantics

Every container has **memory use semantics**:

- "static" costs proportional to container sizes;
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Capacities:

- are **lower bounds**; grb::capacity(s) ≥ 1;
- may **increase** through grb::resize, updates memory use semantics;
- Any request to decrease capacity thus **may be ignored**.

Basics

User I/O:

```
buildMatrixUnique( A, begin_iterator, end_iterator,
  SEQUENTIAL );
buildMatrixUnique( L, i_begin, i_end, j_begin, j_end,
  PARALLEL );
std::cout << "s␣has␣" << grb::nnz( s ) << "␣values:\n";
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User processes: each iterator pair on different processes point to

- the same, complete collection C, leading to **sequential I/O**;
- mutually disjoint collections C_i s.t. $C = \cup_i C_i$: **parallel I/O**.

Performance

Auto-vectorisation versus hand-written code, sequential backend

- dot product, dot(alpha, x, y, semiring)
- reduce, foldl < dense >(alpha, x, associativeOp)
- FMA, eWiseMulAdd< dense >(z, alpha, x, y, semiring)

	Ivy Bridge			Cascade Lake		
	Dot product	Reduction	FMA	Dot product	Reduction	FMA
Hand-coded	120 (12.4)	106 (7.03)	**199** (11.2)	227 (6.56)	221 (3.37)	**216** (10.3)
ALP/GraphBLAS	120 (12.4)	106 (7.03)	204 (11.0)	**226** (6.59)	**220** (3.39)	217 (10.3)
eWiseLambda	125 (12.0)	131 (5.69)	205 (10.9)	228 (6.54)	226 (3.30)	217 (10.3)

Microbenchmarks evaluating ALP/GraphBLAS auto-vectorsation. Figures are in milliseconds (and Gbyte/s).

Theoretical (peak) throughput and approximate throughput per core:

- 190.7 GByte/s; 10 cores per CPU, two CPUs, 9.54 Gbyte/s/core (Ivy);
- 262.2 GByte/s; 22 cores / CPU, 2 CPUs, 5.96 Gbyte/s/core (Cascade).