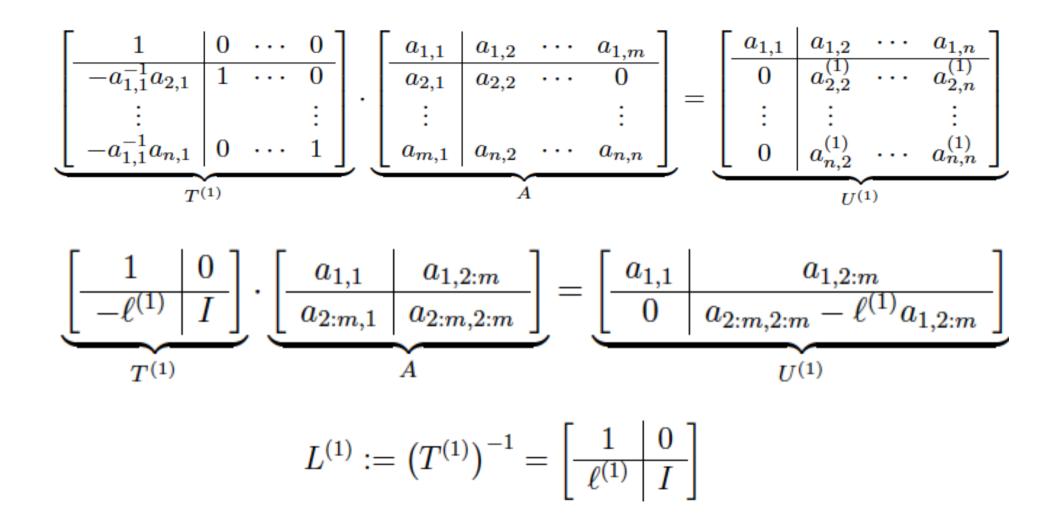
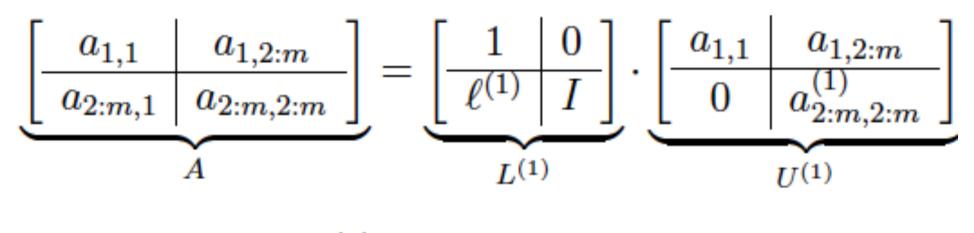
## Derivation of LU factorisation of a matrix A

Inspired by Gaussian elimination

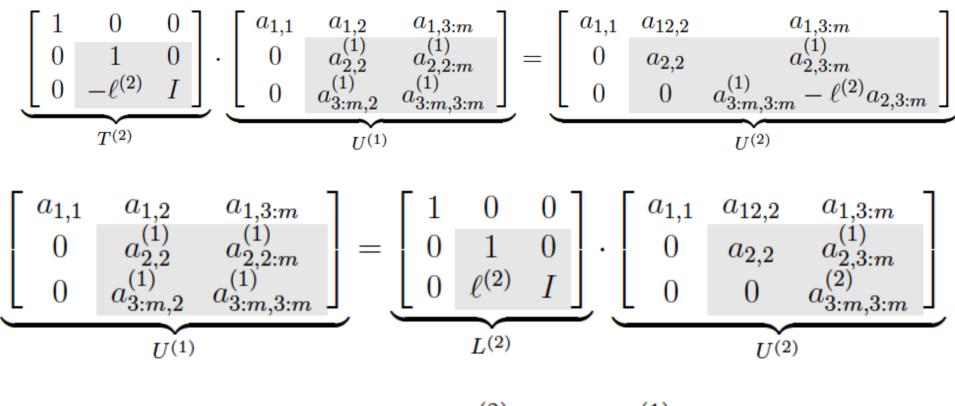
## Gaussian elimination

- Gaussian elimination: transform Ax=b to Ux=y with U upper triangular
- Linear Algebra: there exists a lower triang. transformation matrix Tsuch that TA = U
- Let  $L = T^{-1} \rightarrow L$  is also lower triangular !
- A =  $T^{-1}$
- Note: shift in notation: row & column indices: 1 to n (instead of from 0 to n-1).





 $a_{2:m,2:m}^{(1)} = a_{2:m,2:m} - \ell^{(1)}a_{1,2:m}$ 



$$a_{3:m,3:m}^{(2)} = a_{3:m,3:m}^{(1)} - \ell^{(2)}a_{2,3:m}$$

$$U^{(0)} = A$$
  

$$U^{(k)} = L^{(k+1)}U^{(k+1)} , \quad k = 0, ..., k - 1$$
  

$$U = U^{(n)}$$
  

$$L = L^{(1)} \cdots L^{(n)}$$
  

$$A = LU$$

$$L = L^{(1)} \cdots L^{(n)} = \begin{bmatrix} 1 & 0 \\ \vdots & 1 & 0 \\ \vdots & \vdots & \ddots \\ \ell^{(1)} & \ell^{(2)} & 1 \\ \vdots & \vdots & \ell^{(n-1)} & 1 \end{bmatrix}$$

| Algorithm 2.1. Sequential LU decomposition.  |   |
|--|---|
| input:<br>output:  | $A^{(0)}$ : $n \times n$ matrix.<br>$L$ : $n \times n$ unit lower triangular matrix,<br>$U$ : $n \times n$ upper triangular matrix,<br>such that $LU = A^{(0)}$ . |
| $egin{aligned} 	ext{for } k &:= 0 	ext{ to } n-1 	ext{ do } \ 	ext{for } j &:= k 	ext{ to } n-1 	ext{ do } \ 	ext{} u_{kj} &:= a_{kj}^{(k)}; \ 	ext{for } i &:= k+1 	ext{ to } n-1 	ext{ do } \ 	ext{} l_{ik} &:= a_{ik}^{(k)} / u_{kk}; \ 	ext{for } i &:= k+1 	ext{ to } n-1 	ext{ do } \ 	ext{for } j &:= k+1 	ext{ to } n-1 	ext{ do } \ 	ext{for } j &:= k+1 	ext{ to } n-1 	ext{ do } \ 	ext{} a_{ij}^{(k+1)} &:= a_{ij}^{(k)} - l_{ik} u_{kj}; \end{aligned}$ |   |