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Flynn's Taxonomy

Examples:

- classical von Neumann computing; program and data streamed to processor.
- vectorisation $(MMX/AVX/...)$, vector computers.
- shared-memory programming, i.e.,
	- ccNUMA architectures.
	- symmetric multiprocessors,
	- non-cc NUMA (scratchpad memory, Cell Broadband Engine, . . .)
- distributed-memory programming (classical, grids, clouds, . . .).
- hybrid parallel programming.

Programming models

A class on parallel computing focuses on MIMD.

- Two flavours of MIMD:
	- Single Program, Multiple Data (SPMD), and
	- Multiple Program, Multiple Data (MPMD).
- Both SPMD and MPMD programs can be in **lockstep**.

Examples:

- GPUs often use **stream-based** programming. This is SPMD and employs groupwise lockstepping.
- Past and future architectures do allow and even **promote MPMD** programming: e.g., the Cell BE and the Epiphany archtictures.
- BSP is an SPMD paradigm.

Common HW/OS capabilities

The more common hardware and operating systems support for parallel computation include:

- thread and process execution and control:
	- spawn,
	- abort,
	- wait for.
	- exit.
- mutexes (locks),
- synchronisation barriers,
- **•** transactional memory
	- readily available: atomics,
	- more complex technologies: speculative threading.

In-software equivalents of the above are possible (for instance, the use of spin-locks instead of barriers).

Common design patterns

Often-used techniques for writing parallel software:

- task-based: identify independently executable work and group them into tasks, then schedule these dynamically. This may include modeling inter-task dependencies in terms of input- and output-data. In the extreme: workflows.
- divide-and-conquer: concurrently conquer a divided problem.
- **•** geometric decomposition: often used in spatial computational science simulations (e.g., mesh-based computations).
- pipelining: like task-based parallelism, but with serial-dependencies; this pattern sometimes appears in a subsection of a parallel program.
- master/worker: one process assignes work others.
- fork/join: after a fork a new process appears, starting at the same part of the program.

Task-based parallelisation

A task has three states: (1) wait for data, (2) ready, and (3) done. If \mathcal{T}_i is the set of tasks in state *i*, then a simple scheduler runs a loop similar to:

- While \mathcal{T}_2 is empty, wait.
- Remove a task t from \mathcal{T}_2 .
- Execute t.
- For all tasks $s \in \mathcal{T}_1$, if s waits on t,
- let s know t is done, and \bullet
- if s has no further waits, remove it from \mathcal{T}_1 and add it to \mathcal{T}_2 . \bullet
- Insert t into \mathcal{T}_3 .

This requires thread-safe datastructures (concurrent removals, insertions). The orange statement is critical to performance.

The analysis of a fine-grained (task-based) algortihm differs from what we have seen previously. Steps to perform the analysis:

- view the collection of tasks the program creates as a rooted directed graph $G = (V, E)$; V contain all tasks, $r \in V$ is the initial process of the algorithm, and an edge $e = (v, w) \in E$ indicates w depends on v.
- assign vertex weights $w(v)$ to vertices $v \in V$ such that $w(v)$ equals the amount of work of the task v.

Remark: the sequential running time $T_\mathsf{seq} = \sum_{v \in V} w(v)$.

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Remark: let $P \subset V$ be a path from r to a **leaf vertex** x s.t.

$$
\forall v \in V, \quad \nexists (x, v) \in E,
$$

then $w(P) = \sum_{v \in P} w(v)$ is the **path weight**. The weighted **critical path** P_c has for all other possible paths P that $w(P_c) \geq w(P)$.

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- find the weight of the weighted critical path (and try to minimise this for scalability).

Remark: if $p \to \infty$, the minimum amount of computation remains $w(P_c)$, always.

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Remark: if $p \to \infty$, the minimum amount of computation remains $w(P_c)$, always. The parallel compute time T_p is bounded by

$$
T_p = \mathcal{O}(w(P_c)/r + T_{seq}/p),
$$

with r the computation speed of a single worker.

Fine-grained parallel computing

Decompose a problem into many small tasks, that run concurrently (as much as possible). A run-time scheduler assigns tasks to processes.

- What is small? Grain-size.
- Performance model? Parallelism and span.

Algorithms can be implemented as graphs either explicitly or implicitly:

- Intel: Threading Building Blocks (TBB),
- **OpenMP** (remember the shared-memory SpMV example),
- \bullet Intel / MIT / Cilk Arts: Cilk,
- Google: Pregel,
- \bullet ...

Only two parallel programming primitives: \bullet (binary) fork, and **2** (binary) join. Example: calculate x_4 from $x_n = x_{n-2} + x_{n-1}$ given $x_0 = x_1 = 1$: int f(int $n \searrow$ if(n $\pm=0$ --- n $\pm=1$) return 1; int $x1 = c$ ilk spawn f(n-1); //fork int $x2$ = cilk_spawn f(n-2); //fork cilk sync; $\#$ join return $\times 1 + x^2$; } int main() $\{$ int $x4 = f(4)$; printf("x_4 = %d\n", $&\times 4$); return 0; }

Only two parallel programming primitives:

- \bullet (binary) fork, and
- ² (binary) join.

Definition (Overhead)

The **overhead** of parallel computation is any extra effort expended over the original amount of work T_{seq}

$$
T_o = pT_p - T_{seq}.
$$

The parallel computation time can be expressed in T_o :

$$
T_p = \frac{T_{\text{seq}} + T_o}{p}.
$$

Only two parallel programming primitives:

- \bullet (binary) fork, and
- **2** (binary) join.

Spawned function calls are assigned to one of the available processes by the Cilk run time scheduler.

The Cilk scheduler guarantees, under some assumptions on the determinism of the algorithm, that

 $\mathcal{T}_{o}=\mathcal{O}(\rho\mathcal{T}_{\infty}),$

 $\mathcal{O}(T_1/p + T_{\infty})$

resulting in a parallel run-time T_p bounded by

Coarse-grained parallisation: communicating processes

Popular ways for inter-process communication:

- message passing (MPI, BSP),
- direct remote memory access (BSP, MPI),
- collectives (MPI),
- \bullet global arrays / global address space (GA/PGAS/UPC).

These approaches can be

- Blocking or non-blocking,
- \bullet One-sided or two-sided (and up to p-sided).

Classically,

- MPI is a two-sided blocking message passing (send/receive pairs) paradigm, while
- BSP is a one-sided non-blocking direct remote memory access paradigm.

Naturally, both paradigms have evolved beyond this strict classification.

History of MPI

- 1994: Message Passing Interface (MPI) became available as a standard interface for parallel programming in C and Fortran 77.
- Designed by a committee called the MPI Forum consisting of computer vendors, users, computer scientists.
- Based on sending and receiving messages by a pair of processors. One processor sends; the other receives. Both are active in the communication.
- Underlying model: communicating sequential processes (CSP) proposed by Hoare in 1978.
- MPI itself is not a model. BSP is a model.
- MPI is an interface for a communication library, like BSPIib.

Recent history of MPI

- 1997: MPI-2 standard defined. Added functionality:
	- one-sided communications (put, get, sum)
	- dynamic process management
	- parallel input/output
	- languages $C++$ and Fortran 90
- 2003: first full implementations of MPI-2 arrive, namely MPICH (Argonne National Labs) and LAM/MPI (Indiana University).
- 2004–: Open MPI. Open-source project, merges 3 MPI implementations: LAM/MPI, FT-MPI (University of Tennessee), LA-MPI (Los Alamos National Laboratory).
- Many users still use MPI-1, particularly its latest version MPI-1.3.
- 2012 MPI-3. Major update. More one-sided communications, nonblocking collective communications.

Why use MPI?

- It is available on almost every parallel computer, often in an optimised version provided by the vendor. Thus MPI is the most portable communication library.
- Many libraries are available written in MPI, such as the numerical linear algebra library ScaLAPACK.
- You can program in many different ways using MPI, since it is highly flexible.

Why not?

- It is huge: the full standard has about 300 primitives. The user has to make many choices.
- It is not so easy to learn. Usually one starts with a small subset of MPI. Full knowledge of the standard is hard to attain.
- The one-sided communications of MPI-2 and MPI-3 are rather complicated. If you like one-sided communications you may want to consider BSPlib as an alternative.

Ping pong benchmark

 \bullet The cost of communicating a message of length *n* is

```
T(n) = t_{\text{startup}} + nt_{\text{word}}.
```
Here, t_{startup} is a fixed startup cost and t_{word} is the additional cost per data word communicated.

- Communication of a message (in its blocking form) synchronises the sender and receiver. This is pairwise synchronisation, not global.
- Parameters t_{startup} and t_{word} are usually measured by sending a message from one processor to another and back: ping pong.
- The message length is varied in the ping pong benchmark.
- There is only one ping pong ball on the table.

Send and receive primitives

```
if (s == 2)MPI_Send(x,5,MPI_DOUBLE,3,0,MPI_COMM_WORLD);
if (s==3)
 MPI_Recv(y,5,MPI_DOUBLE,2,0,MPI_COMM_WORLD,
           &status);
```
- Processor $P(2)$ sends 5 doubles to $P(3)$.
- \bullet $P(2)$ reads the data from its array x. After transmission, $P(3)$ writes these data into its array y.
- The integer '0' is a tag for distinguishing between different messages from the same source processor to the same destination processor.
- MPI Send and MPI Recv are of fundamental importance in MPI.

Communicator: the whole processor world

if $(s == 2)$

MPI_Send(x,5,MPI_DOUBLE,3,0,MPI_COMM_WORLD); if $(s==3)$ MPI_Recv(y,5,MPI_DOUBLE,2,0,MPI_COMM_WORLD, &status);

- A communicator is a subset of processors forming a communication environment with its own processor numbering.
- MPI COMM WORLD is the communicator consisting of all the processors.

Send/Receive considered harmful

- 1968: Edsger Dijkstra, guru of structured programming, considered the Go To statement harmful in sequential programming.
- Go To was widely used in Fortran programming in those days. It caused spaghetti code: if you pull something here, something unexpected moves there.
- No one dares to use Go To statements any more.
- Send/Receive in parallel programming has the same dangers, and even more, since several diners eat from the same plate.
- Pull here, pull there, nothing moves: deadlock.
- Deadlock may occur if $P(0)$ wants to send a message to $P(1)$, and $P(1)$ to $P(0)$, and both processors want to send before they receive.

Inner product program mpiinprod

```
int main(int argc, char **argv){
    int p, s, n;
   MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&p);
    MPI_Comm_rank(MPI_COMM_WORLD,&s);
    if (s == 0) {
        printf("Please enter n:\n");
        scanf("%d",&n);
        if(n<0)MPI_Abort(MPI_COMM_WORLD,-1);
    }
    MPI_Bcast(&n,1,MPI_INT,0,MPI_COMM_WORLD);
```
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Collective communication: broadcast

MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);

MPI_Bcast(buf,count,datatype,root,communicator);

- Broadcast count data items of a certain datatype from processor root to all others in the communicator, reading from location buf and also writing it there.
- All processors of the communicator participate.
- Extensive set of collective communications available in MPI. Using these reduces the size of program texts, and allows hardware vendors to provide optimised algorithms.

Inner product program mpiinprod (cont'd)

```
nl= nloc(p,s,n); //local vector length
x = vecallocd(nl); //allocate and initialise x
for (i=0; i \le n]: i++) {
   iglob= i*p+s;
   x[i] = iglob+1;}
```

```
MPI_Barrier(MPI_COMM_WORLD); // global sync for timing
time0=MPI_Wtime(); // wall clock time
```


...

...

Inner product program mpiinprod (cont'd)

```
//do calculate inner product
alpha = mplip(p,s,n,x,x);
```

```
//sync for timing
MPI_Barrier(MPI_COMM_WORLD);
time1=MPI_Wtime();
```

```
...
MPI_Finalize();
exit(0):
```


...

Inner product function mpiip

```
double mpiip(int p,int s,int n,
             double *x,double *y){
    double inprod, alpha;
    int i;
    inprod= 0.0;
    for (i=0; i < nloc(p, s, n); i++)invrod += x[i]*y[i];MPI_Allreduce(&inprod,&alpha,1,MPI_DOUBLE,
                  MPI_SUM,MPI_COMM_WORLD);
```

```
return alpha;
```
}

Collective communication: reduce

MPI_Allreduce(&inprod, &alpha, 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);

MPI_Allreduce(sendbuf, recvbuf, count, datatype, operation, communicator);

- The reduction operation by MPI_Allreduce sums the double-precision local inner products inprod, leaving the result alpha on all processors.
- One can also do this for an array instead of a scalar, by changing the parameter 1 to the array size count, or perform other operations, such as taking the maximum, by changing MPI_SUM to MPI_MAX.

Collectives

More collective operations:

- broadcast,
- \bullet scatter.
- gather,
- reduction (also reduce-scatter),
- all-gather, and
- all-to-all.

For a nice graphical overview, see

<www.mpi-forum.org/docs/mpi-1.1/mpi-11-html/node64.html>

- MPI directly integrates these patterns by providing primitives: MPI_BCAST, MPI_GATHER, MPI_SCATTER, ...
- MPI also supports collectives on vectors instead of single entities (MPI_GATHERV. MPI_SCATTERV, ...), thus enabling optimisations such as the two-phase broadcast.

MapReduce

Google Pregel is a successor of MapReduce, a parallel framework that operates on **large data sets** of key-value pairs

 $S \subseteq K \times V$.

with K a set of possible keys and V a set of values.

MapReduce defines two operations on S:

map: $V \rightarrow \mathcal{P}(K \times V)$:

reduce: $\mathcal{P}(K \times V) \rightarrow K \times V$.

• The map operation is **embarrasingly parallel**: every key-value pair is mapped to a new key-value pair, an entirely local operation.

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- The map operation is **embarrasingly parallel**: every key-value pair is mapped to a new key-value pair, an entirely local operation.
- \bullet The reduction reduces all pairs in S that (may) have the same key, into one single key-value pair: global communication.

MapReduce

Calculating $\alpha = x^T y$ using MapReduce:

Let
$$
S = \{(0, \{x_0, y_0\}), (1, \{x_1, y_1\}), \ldots\}.
$$

- **4** Map: for each pair $(i, \{a, b\})$ write (partial, $a \cdot b$). Applying this map adds $\{(\text{partial}, x_0, y_0), (\text{partial}, x_1, y_1), \ldots\}$ to S.
- ² Reduce: for all pairs with key 'partial', combine their values by addition and store the result using key α . Applying this reduction adds $(\alpha, \sum_{i=0}^{n-1} x_i y_i)$ to S.
- \bullet Done: S contains a single entry with key α and value $\mathsf{x}^\mathcal{T} \mathsf{y}.$

The set S is safely stored on a **resilient file system** to cope with hardware failures.

Pregel

Consider a graph $G = (V, E)$. Graph algorithms may be phrased in an SPMD fashion as follows:

- For each vertex $v \in V$, a thread executes a user-defined SPMD algorithm;
- each algorithm consists of successive local compute phases and global communication phases;
- \bullet during a communication phase, a vertex v can only send messages to $N(v)$, where $N(v)$ is the set of neighbouring vertices of v; i.e., $N(v) = \{w \in V \mid \{v, w\} \in E\}.$

MapReduce and Pregel are variants of the BSP algorithm model!

- a type of fine-grained BSP.
- parallelism in Pregel is slightly odd to think about;
	- e.g., what does its compute graph look like?

Multi-BSP

Multi-BSP is the recursive application of the BSP model.

- BSP: a computer consists out of p CPUs/processors/cores/...
- Multi-BSP: a computer consists out of
	- **1** p other Multi-BSP subcomputers (recursively), or
	- \bullet p units of execution (leaves).
- Each Multi-BSP computer:
	- connects its subcomputers or leaves via a network, and
	- provides local memory.

Reference:

Valiant, Leslie G. "A bridging model for multi-core computing." Journal of Computer and System Sciences 77.1 (2011): 154- 166.

Multi-BSP computer model

CPUs, memory, network.

Multi-BSP computer model

CPUs, memory, network.

Multi-BSP computer model

CPUs, memory, network.

Multi-BSP computer model

A BSP computer $(p_0, g_0, l_0, M_0) \cdots (p_{L-1}, g_{L-1}, l_{L-1}, M_{L-1})$.

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Multi-BSP computer model

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Multi-BSP computer model

Multi-BSP algorithm model

This change of computer model changes the algorithmic model:

- only **local communication** allowed using the local (l_k, g_k) .
- local memory requirements do not exceed the **local memory** M_k ,
- \bullet 'local' is given by the current tree level k.

Multi-BSP cost model

We require extra notation:

- \bullet L: number of **levels** in the tree.
- N_i : number of supersteps on the *i*th level,
- $h_{k,i}$: the maximum of all h-relations within the *i*th superstep on level k .
- $w_{k,i}$: the maximum of all work within the *i*th superstep on level k.

The decomposability of Multi-BSP algorithms, just as with the 'flat' BSP model, again results in a transparent cost model:

$$
T = \sum_{k=0}^{L-1} \left(\sum_{i=0}^{N_k-1} w_{k,i} + h_{k,i} g_k + I_k \right).
$$

Half-time summary

- Multi-BSP is a better model for modern parallel architectures. It closely resembles
	- contemporary shared-memory multi-socket machines,
	- multi-level shared and private cache architectures, and
	- multi-level network topologies (e.g., fat trees).
- Hierarchical modeling also has **drawbacks**. It is more difficult to
	- prove optimality of hierarchical algorithms, and
	- **portably** implement hierarchical algorithms.

Would you like to:

- prove optimality of an algorithm in 16 parameters?
- develop algorithms for a four-level machine?

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Multi-BSP can actually simplify these issues!

Multi-BSP: broadcasting

Memory embedding (shared address space):

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Multi-BSP: broadcasting

Optimal algorithm with embedding (shared address space):

Start SPMD section, entry at leaf level, leaf 6 is source.

Multi-BSP: broadcasting

Optimal algorithm with embedding (shared address space):

On this local BSP computer, communicate val to PID 0 and move up.

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Multi-BSP: broadcasting

Optimal algorithm with embedding (shared address space):

On this upper level, send val to PID 0, **broadcast**, and **move down**.

Multi-BSP: broadcasting

Optimal algorithm with embedding (shared address space):

On this level, only PID 0 has val; **broadcast**, and done.

Multi-BSP: broadcasting

Optimal algorithm with embedding (shared address space):

On this level, only PID 0 has val; broadcast, and done.

Multi-BSP: broadcasting

Input: *val* (a value or \emptyset), Output: the value that was broadcast. if val $\neq \emptyset$ then $set source = true$ else set source $=$ false while not on the Multi-BSP root do if source and $bsp_pid() \neq 0$ then send *val* to PID 0 move upwards in the Multi-BSP tree while not on a leaf node do if $bsp_pid() = 0$ then for $k = 1$ to bsp_nprocs() do send val to PID k move downwards in the Multi-BSP tree return val

Summary

We have seen

- various ways on exploiting parallelism on current hardware,
- a high-level overview of parallel programming paradigms,
- common design patterns used within these paradigms,
- what the hardware and OS can do for you,
- an overview of communication paradigms,
- how to analyse fine-grained parallel applications,
- a short introduction to the Message Passing Interface, and programming using MPI, and
- what the future of BSP might look like.

