Introduction to the Bulk Synchronous Parallel model

Albert-Jan N. Yzelman

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Derived from the book slides by Prof. dr. Rob H. Bisseling, found at

`www.math.uu.nl/people/bisseling/Education/PA/pa.html`

Motivation

Why?

Some problems are too **large** to be solved on one machine

- Google pageranking,
- climate modelling,
- structural stability (skyscrapers, air planes, ...),
- financial market pricing,
- movie rendering,
- ...

Bottlenecks can be time-to-solution, memory requirements, or both.

Why?

Sequential performance has **stalled**

Moore's Law: the number of transistors that one can cost-effectively place on a unit surface, doubles every 1.5 years.

Corollary: processor speeds double every 1.5 years

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Corollary: processor speeds double every 1.5 years

This corollary **broke down** around 2007 due to power unscalability.

Solutions:

- Multi-core processors, optionally using
- slower clock speeds.

Software has to become parallel to keep up with Moore's Law!

Why not?

There are also reasons for not going parallel:

- writing parallel code is more difficult:
  - work distribution,
  - communication minimisation,
  - conflicts due to concurrency.
- The many different parallel architectures complicates things: parallel code may run fast on one architecture, but surprisingly slow on others.

To mitigate these issues, we should program

- **portably**, and
- **transparently**.

How?

There are many different

- architectures (RISC, GPUs, x86, ...),
- network interconnects (Hypercube, ring, fat tree, ...),
- vendors (Intel, Cray, IBM, Oracle, Cisco, ...),
- programming interfaces.

We would like **one standard** model to design for, both for

- algorithm design, and
- hardware design.

How?

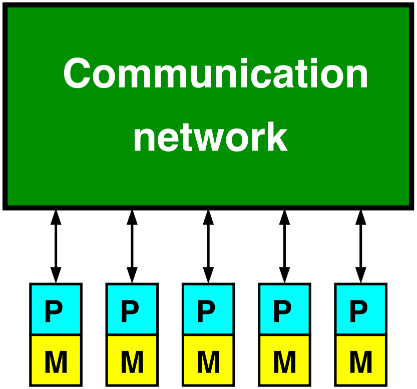
Solution: **bridging models**

- Message Passing Interface (MPI, interface only)
- **Bulk Synchronous Parallel** (BSP, model **and** interface)

Ref.: Leslie G. Valiant, *A bridging model for parallel computation*, Communications of the ACM, Volume 33 (1990), pp. 103–111.

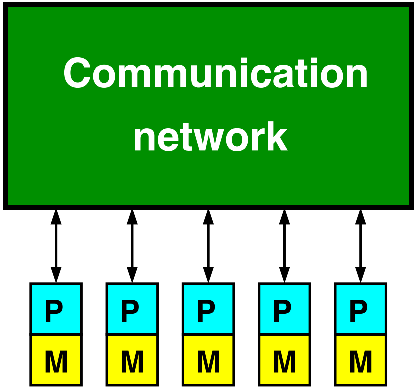
The Bulk Synchronous Parallel Model

BSP computer: abstract model



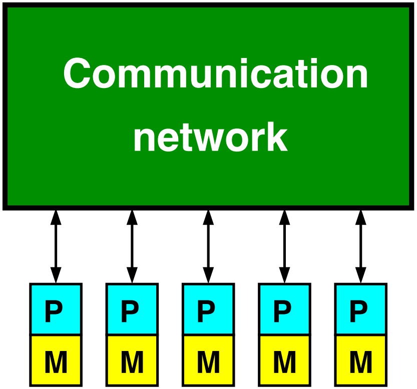
Bulk synchronous parallel (BSP) computer.

BSP computer: abstract model



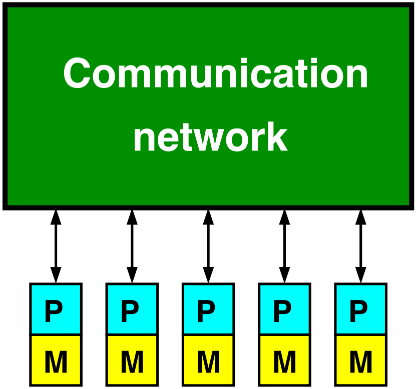
- A BSP computer consists of a collection of processors, each with its own memory. It is a distributed-memory computer.

BSP computer: abstract model



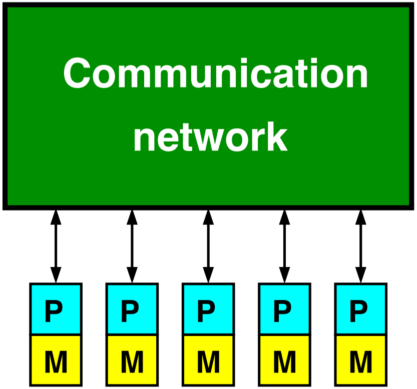
- Access to own memory is fast, to remote memory slower.
- Uniform access time to all remote memories.

BSP computer: abstract model



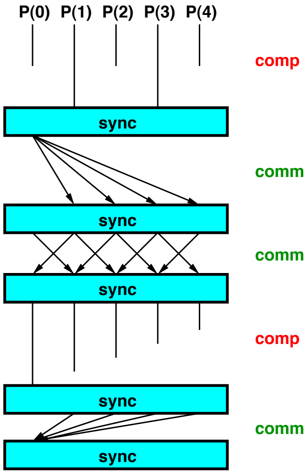
- A **black-box** communication network. Algorithm designers should not worry about network details, only about global performance.

BSP computer: abstract model

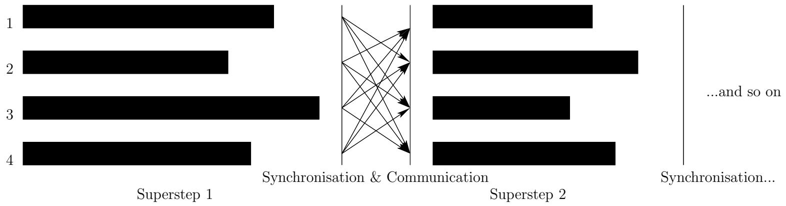


- Algorithms designed for a BSP computer are portable: they can be run efficiently on many different parallel computers.

BSP algorithm

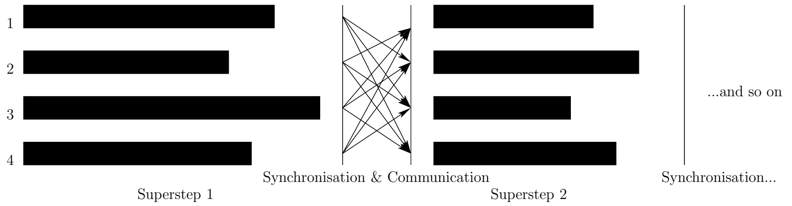


BSP algorithm



- A BSP algorithm consists of
  - **computation** supersteps, and
  - **communication** supersteps.
- A superstep is always followed by a BSP synchronisation barrier.
- A computation superstep is a **sequential** program. In scientific computing, its cost often is expressed in **floating point operations** (flops).

BSP algorithm



- A communication superstep consists of communication operations that **transfer data words** from one processor to another.
- In theory, we distinguish between the two types of supersteps. This **helps** in the **design and analysis** of parallel algorithms.
- In practice the distinction may be dropped, either automatically or manually, so to **overlap communication with computation**.

Cost of a communication superstep

Suppose:

- $h_{\rm s}$ is the maximum number of data words sent by a processor.
- $h_{\rm r}$ is the maximum number of data words received by a processor.

An **h-relation** is a communication superstep in which every processor sends and receives at most h data words:

$$h = \max\{h_{\rm s}, h_{\rm r}\}.$$

Cost of a communication superstep

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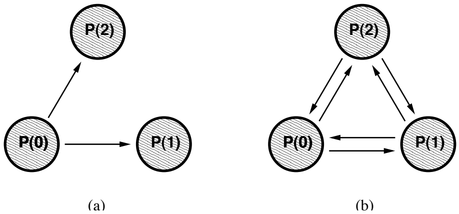
2-relations:



Cost of a communication superstep

Central assumption:

The entry points and exit points of a communication superstep is the bottleneck of the time spent communicating.

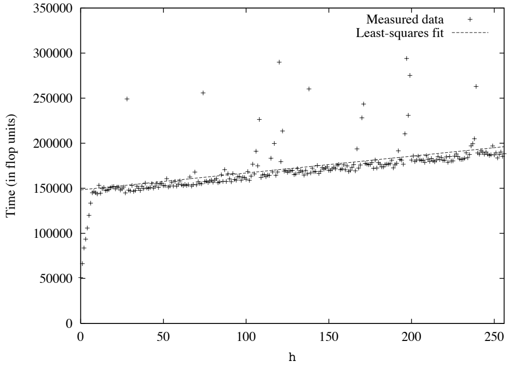
Hence the h-relation determines the cost of a communication superstep:

$$T_{\text{comm}} = hg + l,$$

where g is the time per data word and l the global synchronisation time.

Motivation l: this latency models the cost of preparing the processors and network interconnect for all-to-all communication. This includes start-up costs of sending data, checking whether all data have arrived at their destinations, costs of the employed synchronisation method, ...

Time of an *h*-relation on an 8-processor IBM SP2



$r = 212$ Mflop/s, $p = 8$, $g = 187$ flop (0.88μs),
$l = 148212$ flop ($698\ \mu$s)

Cost of computation superstep

The cost of a computation superstep:

- $T_{\text{comp}} = w/r + l$, where w is the **maximum number of flops** of a processor in the computation superstep.
- Processors with less than w flops will typically be **idle**.
- To measure T, a **wall clock** that measures elapsed time is needed. Using a CPU timer will not work, since it ignores idle time.
- Here, r is measured in **flops per second** (flop/s).

The cost of a BSP algorithm is the sum of the cost of its supersteps:

$$T_{\text{comm}} + T_{\text{comp}} = w + gh + 2l.$$

BSP cost

The cost of a BSP algorithm is an expression of the form

$$a + bg + cl.$$

This cost is obtained by adding the costs of all the supersteps.

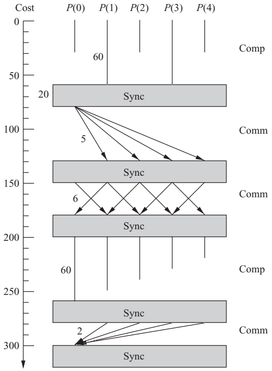
- Note that $g = g(p)$ and $l = l(p)$ are in general a function of the number of processors p.
- The parameters a, b, c depend in general on p and on a problem size n.

In general, the total BSP cost equals:

$$\sum_{i=0}^{N-1} \max w_i^{(s)}/r + h_i g + l,$$

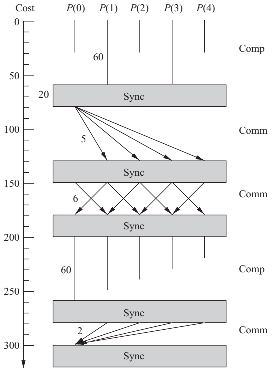
with N the total number of supersteps of the BSP algorithm.

BSP cost



For $p = 5$, $g = 2.5$, and $l = 20$:
First computation superstep costs $60 + 20 = 80$ flops.
First communication superstep costs $4 \cdot 5 \cdot 2.5 + 20 = 70$ flops.

BSP cost



For $p = 5$, $g = 2.5$, and $l = 20$:

The total cost of the BSP algorithm is 320 flops.

Summary

An abstract BSP machine is a BSP(p, r, g, l) computer, with

p	number of processors
r	computing rate (in flop/s)
g	communication cost per data word (in flop time units)
l	global BSP latency cost (in flop time units)

The BSP model consists of

1. a distributed-memory architecture with a black box communication network providing uniform access time to remote memories;
2. an algorithmic framework formed by a sequence of supersteps;
3. a cost model giving cost expressions of the form $a + bg + cl$.

Parallel inner-product computation

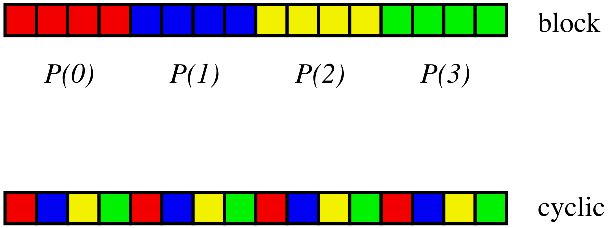
Inner product of two vectors

The inner product of two vectors $\mathbf{x} = (x_0, \ldots, x_{n-1})^{\mathrm{T}}$ and $\mathbf{y} = (y_0, \ldots, y_{n-1})^{\mathrm{T}}$ is defined by

$$\alpha = \mathbf{x}^{\mathrm{T}}\mathbf{y} = \sum_{i=0}^{n-1} x_i y_i.$$

Here, 'T' denotes transposition. All vectors are column vectors.

Vector data distributions



p = #processors = 4

n = vector length = 16

Block distribution

- The block distribution is defined by

$$x_i \longmapsto P(i \mathrm{\ div\ } b), \text{ for } 0 \leq i < n.$$

  Here, the div operator stands for dividing and rounding down: $i \mathrm{\ div\ } b = \lfloor i/b \rfloor$.
- The block size is $b = \lceil \frac{n}{p} \rceil = \frac{n}{p}$ rounded up.
- For $n = 9$ and $p = 4$, this assigns 3, 3, 3, 0 vector components to the processors, respectively. You may blink at an empty processor, but this distribution is just as good as 3, 2, 2, 2. Really!

Cyclic distribution

The cyclic distribution is defined by

$$x_i \longmapsto P(i \bmod p), \text{ for } 0 \leq i < n.$$

This distribution is easiest to compute. Note the advantage of starting to count at zero: the formula becomes very simple.

Parallel inner product computation

Design pattern:

- Assign data so that the bulk of the computations are **local**.

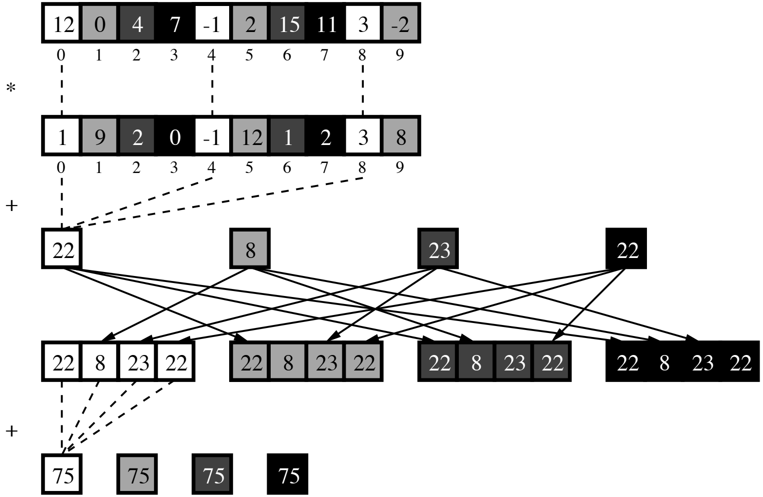
Assign x_i and y_i to the same processor, for all i. This makes computing $x_i \cdot y_i$ a local operation. Thus $\mathrm{distr}(\mathbf{x}) = \mathrm{distr}(\mathbf{y})$.

- Give each process the same amount of work (**load balance**)

Choose a distribution with an even spread of vector components. Both block and cyclic distributions are fine. We choose cyclic, following the way card players deal their cards.

The data distribution naturally leads to a parallel algorithm.

Example for $n = 10$ and $p = 4$



Parallel inner product algorithm for $P(s)$

input:	$\mathbf{x}, \mathbf{y}$: vector of length n,
	$\mathrm{distr}(\mathbf{x}) = \mathrm{distr}(\mathbf{y}) = \phi$,
	with $\phi(i) = i \bmod p$, for $0 \leq i < n$.
output:	$\alpha = \mathbf{x}^T \mathbf{y}$.

```
(0)   $\alpha_s := 0$;
      for $i := s$ to $n - 1$ step $p$ do
          $\alpha_s := \alpha_s + x_i y_i$;
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          put $\alpha_s$ in $P(t)$;
```

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(1)   for $t := 0$ to $p-1$ do
          put $\alpha_s$ in $P(t)$;

(2)   $\alpha := 0$;
      for $t := 0$ to $p-1$ do
          $\alpha := \alpha + \alpha_t$;
```

Single Program, Multiple Data (SPMD)

- Only one program text needs to be written. All processors run the same program, but on their own data.
- The program text is parametrised in the processor number s, $0 \leq s < p$, also called processor identity. The actual execution of the program depends on s.
- Processor $P(s)$ computes a **local** partial inner product

$$\alpha_s = \sum_{0 \leq i < n,\ i \bmod p = s} x_i y_i.$$

- The corresponding computation superstep (0) costs

$$2 \left\lceil \frac{n}{p} \right\rceil + l.$$

(1 addition and 1 multiplication per local vector component.)

Result needed on all processors

- The partial inner products must be added. This could have been done by $P(0)$, i.e. processor 0.

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- Computing α on $P(0)$ costs the same as computing it on all the processors redundantly, i.e. in a replicated fashion. The cost is $p+l$.
- Often, the result is needed on all processors. An example is iterative linear system solvers. The algorithm does just this.
- Sending the local result to all processors is best if each processor contributes one value. If there are more values per processor, a different approach might be better.

Total BSP cost of inner product

$$T_{\mathrm{inprod}} = 2\left\lceil \frac{n}{p} \right\rceil + p + (p-1)g + 3l.$$

One-sided communication

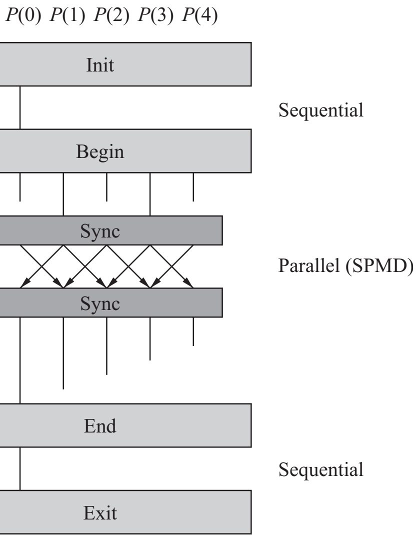
- The ‘put’ operation involves an active sender and a passive receiver. We assume all puts are accepted. Thus we can define each data transfer by giving only the action of **one side**.
- No clutter in programs: **shorter and simpler** texts.
- **No** danger of the dreaded **deadlock**. What happens if both processors want to receive first? Deadlock can easily occur in two-sided message passing, with an active sender and an active receiver that must shake hands, or kiss. This may cause lots of problems.
- Another one-sided operation is the ‘get’. The name says it all.
- One-sided communications are **more efficient**.

Summary

- We design algorithms in Single Program, Multiple Data style. Each processor runs its own copy of the same program, on its own data.
- The block and cyclic distributions are commonly used in parallel computing. Both are suitable for an inner product computation.
- The BSP style encourages balancing the communication among the processors. Sending all data to one processor is discouraged. Better: all to all.
- One-sided communications such as puts and gets are easy to use and efficient.
- The BSP cost is transparently calculated.

BSPlib, the BSP interface

BSPlib program: sequential, parallel, sequential



Sequential I, parallel computation, sequential O

- A BSPlib program starts with a sequential part, mainly intended for input. Motivation:
  - Desired number of processors of the parallel part may depend on the input.
  - Input of data describing a problem is often sequential.
- A BSPlib program ends with a sequential part, mainly intended for output. Motivation:
  - Reporting the output of a computation is often sequential.
- Sequential I/O in a parallel program may be inherited from a sequential program.
- The sequential parts may also be empty.

Main function of BSPlib program

```
int P;
int main(int argc, char **argv){
    bsp_init( bspinprod, argc, argv );

    printf("How many processors?\n"); /* sequential part */
    scanf( "%d", &P );
    if( P > bsp_nprocs() ) {
        printf( "Sorry, not enough available.\n" );
        exit( 1 );
    }

    bspinprod();                          /*  parallel part  */

    exit( 0 );                            /* sequential part */
}
```

Primitive bsp_init

```
bsp_init( spmd, argc, argv );
```

- The BSPlib primitive `bsp_init` initialises the program. It must be the first executable statement in the program.
- `spmd` is the name of the function that comprises the parallel part . In our example, the name is `bspinprod`.
- The primitive `bsp_init` is needed to circumvent restrictions of certain machines.
- It is ugly and often misunderstood. (But then, what happened to Quasimodo in the end?)
- `int argc` is the number of command-line arguments and `char **argv` is the array of arguments. These arguments can be used in the sequential input part, but they **cannot be transferred** to the parallel part.

Structure of SPMD part

```
void bspinprod() {
    int p, s, n;

    bsp_begin( P );
    p = bsp_nprocs(); /* p = number of procs  */
    s = bsp_pid();    /* s = processor number */
    if( s == 0 ) {
        printf( "Please enter n:\n" );
        scanf( "%d", &n );
        if( n < 0 )
            bsp_abort( "Error in input: n < 0" );
    }
    ...
    bsp_end();
```

Primitives bsp_begin, bsp_end

```
bsp_begin( reqprocs );
bsp_end();
```

- The BSPlib primitive `bsp_begin` starts the parallel part of the program with `reqprocs` processors. It must be the first executable statement in the SPMD function.
- The BSPlib primitive `bsp_end` ends the parallel part of the program. It must be the last executable statement in the SPMD function.
- If the sequential parts of the program are empty, `main` can become the parallel part and `bsp_init` can be removed.
- $P(0)$ inherits the values of the variables from the sequential part and can use these in the parallel part. Other processors do not inherit any values and must obtain needed values by explicit communication.

Primitives bsp_nprocs, bsp_pid

```
bsp_nprocs();
bsp_pid();
```

- The BSPlib primitive `bsp_nprocs` gives the number of processors. In the parallel part, this is the actual number p of processors involved in the parallel computation. In the sequential parts, it is the maximum number available.
- Thus, we can ask how many processors are available and then decide not to use them all. **Sometimes, using fewer processors gives faster results!**
- The BSPlib primitive `bsp_pid` gives the processor identity s, where $0 \leq s < p$.
- Both primitives can be used anywhere in the parallel program, so you can always get an answer to burning questions such as: How many are we? Who am I?

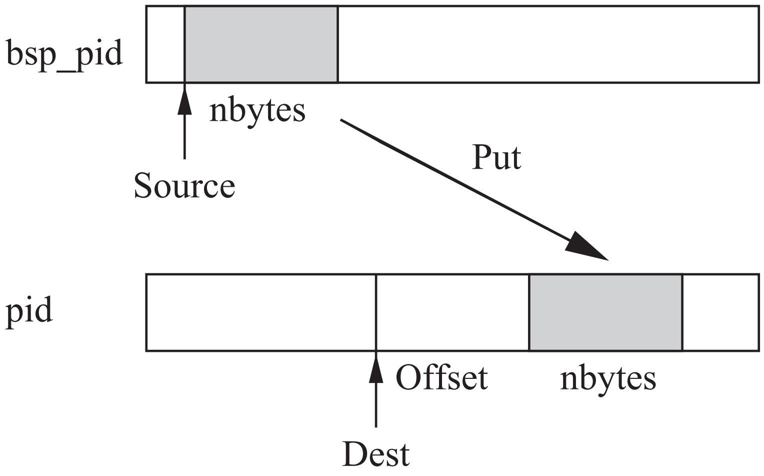
Primitive bsp_abort

```
bsp_abort( error_message );
```

- If one processor detects that something is wrong, it can bring all processors down in a graceful manner and print an error message by using `bsp_abort`.
- The message is in the standard format of the C-function `printf`.

Putting data into another processor

```
bsp_put(pid, source, dest, offset, nbytes);
```



Primitive bsp_put

```
bsp_put(pid, source, dest, offset, nbytes);
```

- The `bsp_put` operation **copies** `nbytes` of data from the local processor `bsp_pid` into the specified destination processor `pid`.
- The pointer `source` points to the start of the data to be copied.
- The pointer `dest` specifies the start of the memory area where the data is written.
- The data is written at `offset` bytes from the start.
- This is the most-often used one-sided communication operation.

Inner product function

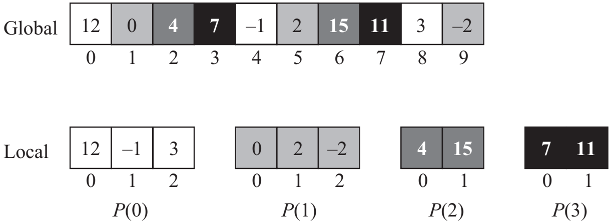
```
double bspip( int p, int s, int n, double *x, double *y ) {

    double inprod, *Inprod;                /* Initialisation */
    int i, t;
    Inprod = vecallocd( p );
    bsp_push_reg( Inprod, p * SZDBL );
    bsp_sync();

    inprod = 0.0;                          /*  Superstep 0   */
    for ( i = 0; i < nloc( p, s, n ); i++ )
        inprod += x[i] * y[i];

    for(t = 0; t < p; t++ )                /*  Superstep 1   */
        bsp_put( t, &inprod, Inprod, s*SZDBL, SZDBL );
    bsp_sync(); ...
```

Local and global indices for cyclic distribution



Global index: i, local index on $P(s)$: $\mathtt{i}$. Their relation is:

$$i = \mathtt{i} \cdot p + s.$$

Use local indices in programs:

```
for( i = 0; i < nloc(p, s, n); i++ )
    inprod += x[i] * y[i];
```

Primitive bsp_get

```
bsp_get(pid, source, offset, dest, nbytes);
```

- The `bsp_get` operation copies `nbytes` of data from the specified remote source processor `pid` into the local processor `bsp_pid`.
- The pointer `source` points to the start of the data in the remote processor to be copied.
- The pointer `dest` specifies the start of the local memory area where the data is written.
- The data is read starting at `offset` bytes from the start of `source`.
- Remember for both puts and gets: the source parameter comes first and the offset is in the remote processor.

Getting *n* from *P*(0)

```
void bspinprod() {

    int p, s, n;
    ...
    if( s == 0 ) {
        printf( "Please enter n:\n" );
        scanf( "%d", &n );
    }
    bsp_push_reg( &n, SZINT);
    bsp_sync();

    bsp_get( 0, &n, 0, &n, SZINT );
    bsp_sync();
    ...
}
```

Primitive bsp_sync

```
bsp_sync();
```

- The bsp_sync operation terminates the current superstep. It causes all communications initiated by puts and gets to be actually carried out. It synchronises all the processors.
- After the bsp_sync, the communicated data can be used.

Safety first: no interference

- The regular `bsp_put` and `bsp_get` operations are doubly buffered, at the source and at the destination. This provides safety.
- A data word that is put is first copied into a local send buffer. The space occupied by the original data word can be reused immediately.
- All received data are first stored in a receive buffer.
- All communication is postponed until the moment all computations of the current superstep are finished. The value obtained by a get is the value at the moment computations are finished; **the get cannot buffer data!**
- If you like living on the edge: the `bsp_hpput` primitive is **unbuffered**, more efficient than `bsp_put`, uses less memory, but is considered dangerous.

Your x is my x

```
bsp_push_reg( variable, nbytes );
bsp_pop_reg( variable);
```

- A variable called `x` may have the **same name** on different processors, but this does not guarantee that it has the **same address** in memory.
- To guarantee this, the names must be **registered** first.
- All processors participate in the registration procedure by pushing their variable and its memory size onto a stack. The unwilling ones can register `NULL`.
- The SPMD style suggests registering the same variable name on all processors, but this is not strictly necessary.
- Registration takes effect only in the next superstep.
- Deregistration is done by all processors together.

Registration is expensive

- To register, all processors have to talk to each other.
- This is a *p*-relation, worst case.
- Try to register sparingly.
- Register once, put many times.

BSP timer measures elapsed time

```
...
bsp_sync();
time0 = bsp_time();

alpha = bspip( p, s, n, x, x );
bsp_sync();
time1=bsp_time();

if( s==0 )
    printf( "This took only %.6lf seconds.\n",
            time1 - time0 );
...
```

Summary

- SMALL IS BEAUTIFUL
- BSPlib is a small library of 20 primitives for writing parallel programs in bulk synchronous parallel style.
- We have learned 12 primitives and are ready to start programming in parallel.
- The put and get primitives provide RDMA (Remote Direct Memory Access, also called DRMA).
- Registration allows direct access to dynamically allocated memory.
- The complete program `bspinprod` should now be clear. Try to compile it and to run it on 4 processors.

Programming using BSPlib

Available BSP libraries

Distributed-memory computers:

1. Oxford BSP library, see `www.bsp-worldwide.org`
2. BSPonMPI, see `bsponmpi.sourceforge.net`

Shared-memory computers, see `www.multicorebsp.com`:

1. MulticoreBSP for Java
2. **MulticoreBSP for C**

Compilation of BSPlib programs

Oxford BSP Toolset (Hill, McColl, Stefanescu, Goudreau, Lang, Rao, Suel, Tsantilas, Bisseling; 1998)

```
bspcc bspinprod.c;
bsprun -npes <P> ./a.out
```

BSPonMPI (van Suijlen; 2006)

```
mpcc bspinprod.c -lbsponmpi;
./a.out
```

MulticoreBSP for C (Yzelman, Bisseling, Roose, Meerbergen; 2012)

```
cc bspinprod.c -lmcbsp -pthread -lrt;
./a.out
```

MulticoreBSP for Java (Yzelman, Bisseling; 2010)

```
javac -cp MulticoreBSP.jar bspinprod.java;
java -cp MulticoreBSP.jar bspinprod
```

Other BSP libraries

Other (distributed memory) BSP-style libraries (incompatible API):

- MapReduce

  Google Inc.; 2004.
- Pregel

  Malewicz, Austern, Bik, Dehnert, Ilan Horn, Czajkowski (Google Inc.); 2010.
- Apache Hama

  Yoon et al.; 2010
- Paderborn University BSP library (PUB)

  Bonorden, Juurlink, von Otte, Rieping; 1998.
- Bulk Synchronous Parallel ML (BSMLlib)

  Gava, Gesbert, Hains, Tesson; 2000.
- Python/BSP

  Hinsen, Sadron; 2003.
- Cloudscale BSP

  McColl et al. (Cloudscale Inc.); 2012.

Hello World!

```
#include "bsp.h" / "mcbsp.h"

int main(int argc, char **argv) {
    bsp_begin( 4 );

    printf( "Hello world from thread %d out of $d!\n",
        bsp_pid(), bsp_nprocs() );

    bsp_end();
}
```

Hello World!

An example using MulticoreBSP for C and static linkage:

```
$ gcc -o hello hello.c lib/libmcbsp1.1.0.a -pthread -lrt
$ ./hello
Hello world from thread 3 out of 4!
Hello world from thread 2 out of 4!
Hello world from thread 0 out of 4!
Hello world from thread 1 out of 4!
$ ...
```

Hello World!

```
#include "mcbsp.h"
int P;

void hello() {
    bsp_begin( P );
    printf( "Hello world from thread %d!\n",
        bsp_pid() );
    bsp_end();
}

int main(int argc, char **argv) {
    bsp_init( hello, argc, argv );
    scanf( "%d", &P );
    hello();
}
```

Summary

- There are multiple implementations of the BSPlib interface.
- Other BSP-style libraries for parallel programming exist; some stay close to the BSP model, others do not.
- We have seen the syntax for compilation for various BSPlib libraries, and applied these on a ‘Hello world!’ example.
- The complete program `bspinprod` should now be clear from the last section. Try to compile it and to run it!