Multi-BSP computing: the next step?

ExaScience life lab

3rd of December 2014

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Outline

The Multi-BSP model

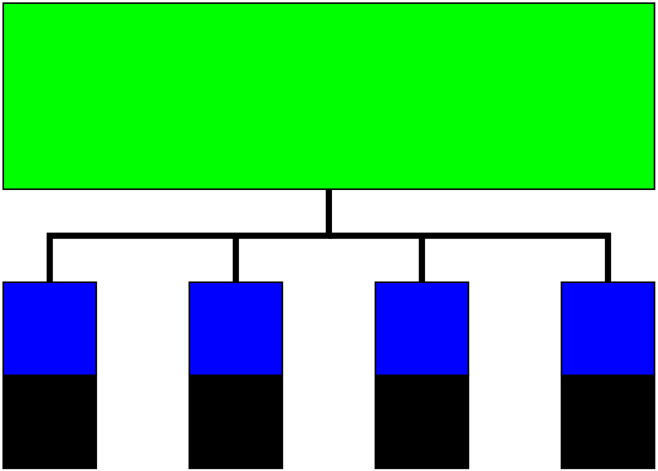
Three concepts

The Bulk Synchronous Parallel

1. computer,
2. algorithm,
3. cost model.

BSP computer

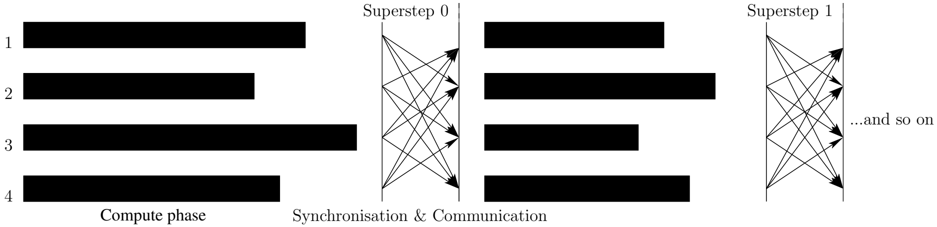
CPUs, memory, network.



A BSP computer (p, g, l).

BSP algorithm

- computations are grouped into **phases**,
- **no communication** during computation,
- communication is only allowed **between** computation phases.



BSP cost model

The **cost of computation** during the ith superstep is

$$T_{\text{comp},i} = \max_s w_i^{(s)}.$$

The total **cost of communication** during the ith superstep is

$$T_{\text{comm,i}} = h_i g.$$

Adding up superstep costs, separated by the latency l, yields the **full BSP cost**:

$$T = \sum_{i=0}^{N-1} \left(T_{\text{comp},i} + T_{\text{comm,i}} + l\right) = \sum_{i=0}^{N-1} \left(\max w_i^{(s)} + h_i g + l\right),$$

Multi-BSP

Multi-BSP is the recursive application of the BSP model.

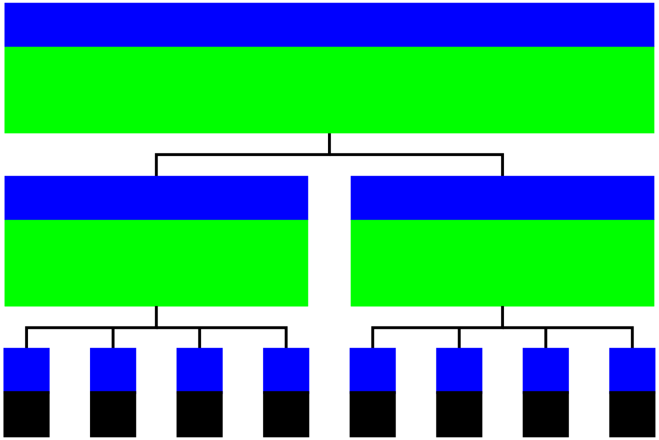
- **BSP**: a computer consists out of p CPUs/processors/cores/...
- **Multi-BSP**: a computer consists out of
  1. p *other* Multi-BSP subcomputers (recursively), **or**
  2. p units of execution (leaves).
- Each Multi-BSP computer:
  - connects its subcomputers or leaves via a network, and
  - provides local memory.

Reference:

Valiant, Leslie G. "A bridging model for multi-core computing." Journal of Computer and System Sciences 77.1 (2011): 154-166.

Multi-BSP computer model

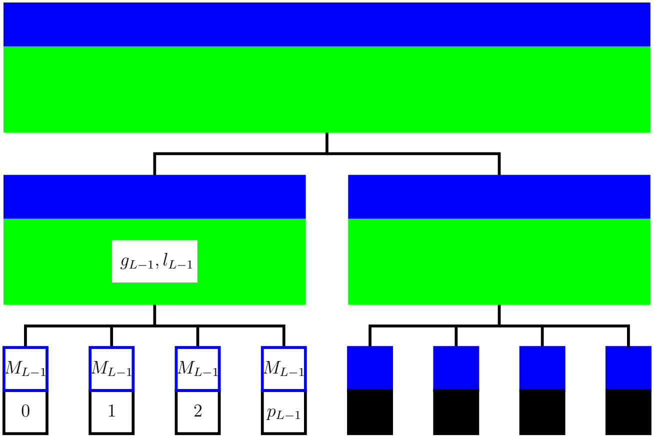
CPUs, memory, network.



A BSP computer $(p_0, g_0, l_0, M_0) \cdots (p_{L-1}, g_{L-1}, l_{L-1}, M_{L-1})$.

Multi-BSP computer model

CPUs, memory, network.



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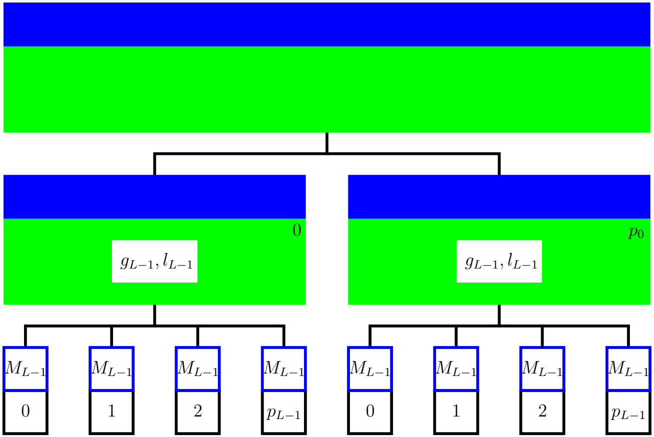

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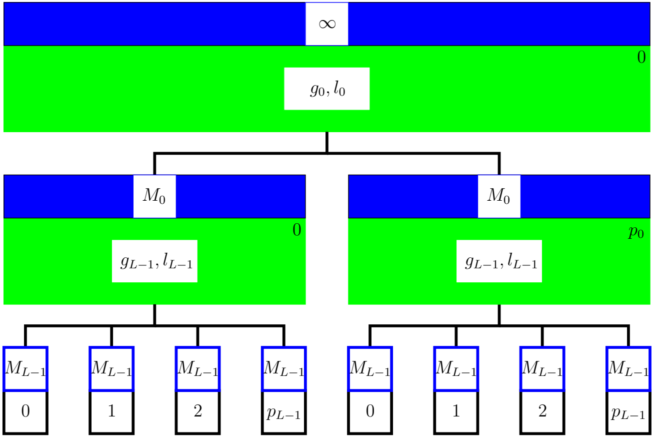

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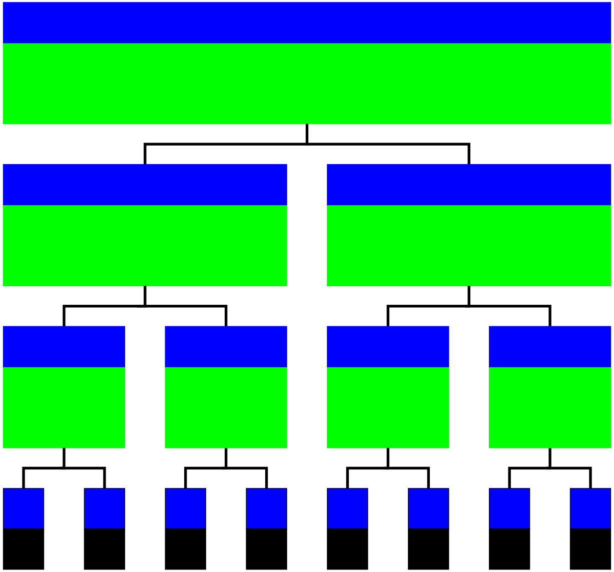

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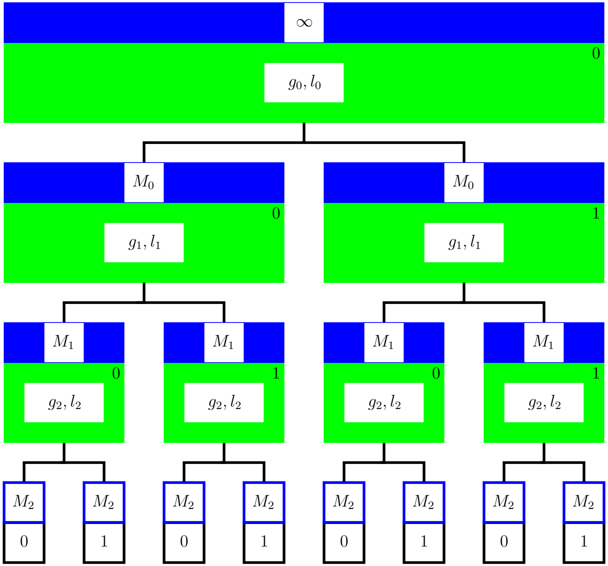
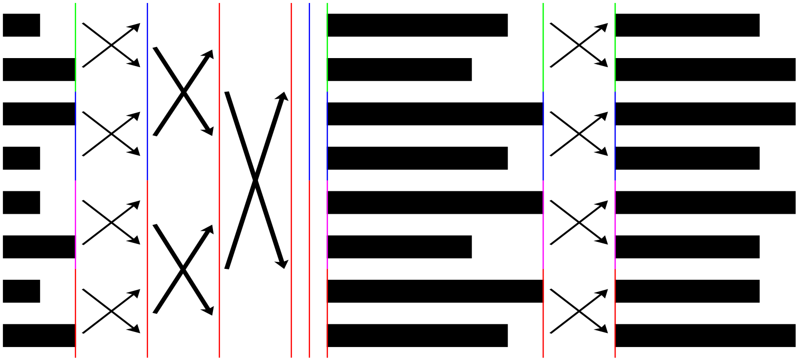


Multi-BSP algorithm model

This change of computer model changes the algorithmic model:

- only **local communication** allowed using the local (l_k, g_k),
- local memory requirements do not exceed the **local memory** M_k,
- 'local' is given by the **current tree level** k.



Multi-BSP cost model

We require extra notation:

- L: number of **levels** in the tree,
- N_i: number of **supersteps** on the ith level,
- $h_{k,i}$: the **maximum of all $\mathbf{h}$-relations** within the ith superstep on level k,
- $w_{k,i}$: the **maximum of all work** within the ith superstep on level k.

The decomposability of Multi-BSP algorithms, just as with the ‘flat’ BSP model, again results in a **transparent cost model**:

$$T = \sum_{k=0}^{L-1} \left(\sum_{i=0}^{N_k - 1} w_{k,i} + h_{k,i} g_k + l_k \right).$$

Half-time summary

- Multi-BSP is **a better model** for modern parallel architectures. It closely resembles
  - contemporary shared-memory multi-socket machines,
  - multi-level shared and private cache architectures, and
  - multi-level network topologies (e.g., fat trees).
- Hierarchical modeling also has **drawbacks**. It is more difficult to
  - **prove optimality** of hierarchical algorithms, and
  - **portably** implement hierarchical algorithms.

Would you like to:

- prove optimality of an algorithm in 16 parameters?
- develop algorithms for a four-level machine?

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- develop algorithms for a four-level machine?

Multi-BSP can actually **simplify** these issues!

Philosophy

Motivation

Reasons for parallel computing:

1. to speed up a **long computation**, or
2. to split up a **huge computation**.

That is,

we wish to scale in time and memory.

Speedup

Definition (Speedup)

Let T_{seq} be the sequential running time required for solving a problem. Let T_p be the running time of a parallel algorithm using p processes, solving the same problem. Then the **speedup** is given by

$$S = T_{\text{seq}}/T_p.$$

Scalable in time:

Ideally,	$S = p$;
if we are lucky,	$S > p$;
realistically,	$1 \ll S < p$;
if we do very badly,	$S < 1$.

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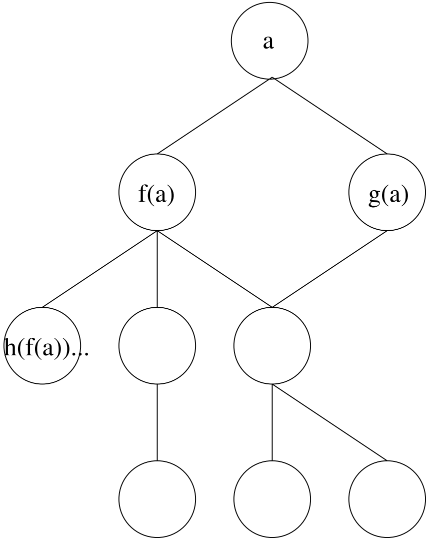
But what is a good speedup?

Maximum attainable speedup

Consider a graph $G = (V, E)$ of a given algorithm, e.g.,



– Nodes correspond to data, edges indicate which data is combined to generate a certain output.

Question: *If we had an infinite number of processors, how fast would we be able to run the algorithm shown on the right?*

Maximum attainable speedup

Consider a graph $G = (V, E)$ of a given algorithm, e.g.,

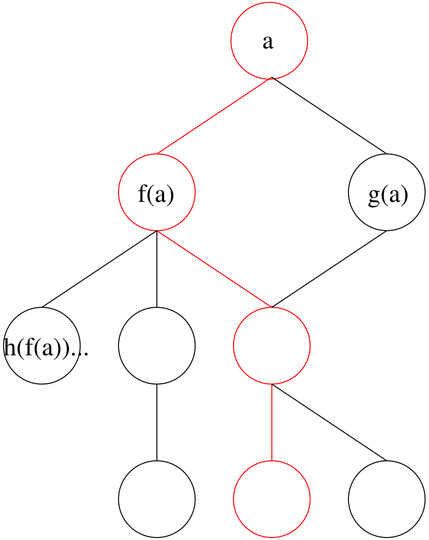
- Nodes correspond to data, edges indicate which data is combined to generate a certain output.

Question: *If we had an infinite number of processors, how fast would we be able to run the algorithm shown on the right?*

Answer: $T_{\text{seq}} = |V| = 9$, while the **critical path length** T_∞ equals 4.

The maximum speedup hence is:

$$T_{\text{seq}}/T_\infty = 9/4.$$



What is parallelism?

Definition (Parallelism)

The **parallelism** of a given algorithm is its maximum attainable speedup:

$$T_{\text{seq}}/T_\infty.$$

T_∞ is known as the **critical path length** or the **algorithmic span**.

This leads to a theoretical **upper bound on speedup**:

$$S = T_{\text{seq}}/T_p \leq T_{\text{seq}}/T_\infty.$$

This type of analysis forms the basis of

fine-grained parallel computation.

Fine-grained parallel computing

Decompose a problem into many small tasks, that run **concurrently** (as much as possible). A **run-time scheduler** assigns tasks to processes.

- What is small? **Grain-size**.
- Performance model? **Parallelism**.

Algorithms can be implemented as graphs either explicitly or **implicitly**:

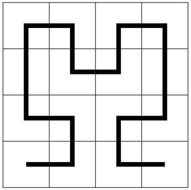
- Intel: Threading Building Blocks (TBB),
- **OpenMP**,
- Intel / MIT / Cilk Arts: **Cilk**,
- Google: **Pregel**,
- ...

By contrast, BSP computing is **coarse-grained**.

OpenMP

Example via SpMV multiplication. Assuming a Hilbert nonzero ordering:

$$A = \begin{pmatrix} 4 & 1 & 3 & 0 \\ 0 & 0 & 2 & 3 \\ 1 & 0 & 0 & 2 \\ 7 & 0 & 1 & 1 \end{pmatrix}$$



COO (triplet) storage:

$$A = \begin{cases} V & [7\ 1\ 4\ 1\ 2\ 3\ 3\ 2\ 1\ 1] \\ J & [0\ 0\ 0\ 1\ 2\ 2\ 3\ 3\ 3\ 2] \\ I & [3\ 2\ 0\ 0\ 1\ 0\ 1\ 2\ 3\ 3] \end{cases}$$

```
for $k = 0$ to $nz - 1$ do
    add $V_k \cdot x_{J_k}$ to $y_{I_k}$
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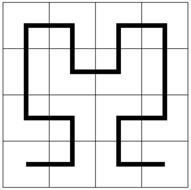

COO (triplet) storage: $T_{\text{seq}}/T_\infty = 2nz$..?

$$A = \begin{cases} V & [7\ 1\ 4\ 1\ 2\ 3\ 3\ 2\ 1\ 1] \\ J & [0\ 0\ 0\ 1\ 2\ 2\ 3\ 3\ 3\ 2] \\ I & [3\ 2\ 0\ 0\ 1\ 0\ 1\ 2\ 3\ 3] \end{cases}$$

```
#omp parallel for private( k ) schedule( dynamic, 8 )
for $k = 0$ to $nz - 1$ do
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OpenMP

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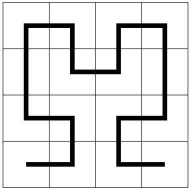

COO (triplet) storage: data race! (concurrent writes to the same y_i)

$$A = \begin{cases} V & [7\ 1\ 4\ 1\ 2\ 3\ 3\ 2\ 1\ 1] \\ J & [0\ 0\ 0\ 1\ 2\ 2\ 3\ 3\ 3\ 2] \\ I & [3\ 2\ 0\ 0\ 1\ 0\ 1\ 2\ 3\ 3] \end{cases}$$

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for $k = 0$ to $nz - 1$ do
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```

OpenMP

Example via SpMV multiplication. Assuming row-major nonzero ordering:

$$A = \begin{pmatrix} 4 & 1 & 3 & 0 \\ 0 & 0 & 2 & 3 \\ 1 & 0 & 0 & 2 \\ 7 & 0 & 1 & 1 \end{pmatrix}$$

CRS storage:

$$A = \begin{cases} V & [4\ 1\ 3\ 2\ 3\ 1\ 2\ 7\ 1\ 1] \\ J & [0\ 1\ 2\ 2\ 3\ 0\ 3\ 0\ 2\ 3] \\ \hat{I} & [0\ 3\ 5\ 7\ 10] \end{cases}$$

```
for $i = 0$ to $m - 1$ do
    for $k = \hat{I}_i$ to $\hat{I}_{i+1} - 1$ do
        add $V_k \cdot x_{J_k}$ to $y_i$
```

OpenMP

Example via SpMV multiplication. Assuming row-major nonzero ordering:

$$A = \begin{pmatrix} 4 & 1 & 3 & 0 \\ 0 & 0 & 2 & 3 \\ 1 & 0 & 0 & 2 \\ 7 & 0 & 1 & 1 \end{pmatrix}$$

CRS storage: $T_{\text{seq}}/T_\infty = nz/\max_i |a_{i:}|$ (with $a_{i:}$ the ith row of A)

$$A = \begin{cases} V & [4\ 1\ 3\ 2\ 3\ 1\ 2\ 7\ 1\ 1] \\ J & [0\ 1\ 2\ 2\ 3\ 0\ 3\ 0\ 2\ 3] \\ \hat{I} & [0\ 3\ 5\ 7\ 10] \end{cases}$$

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#omp parallel for private( i, k ) schedule( **dynamic**, 8 )
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```
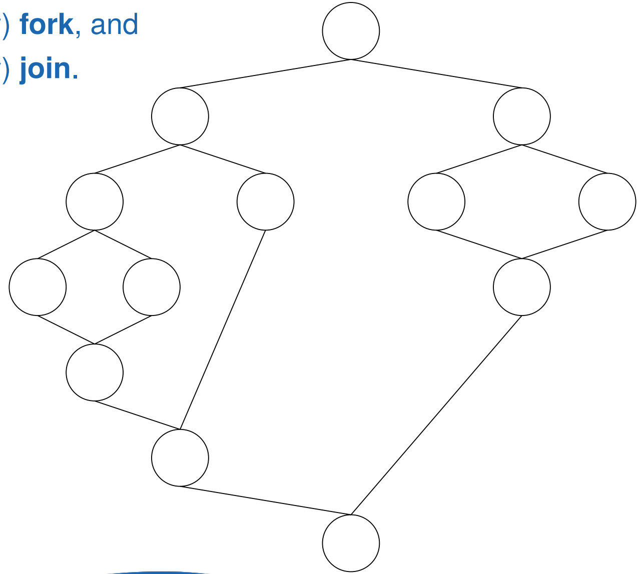

Cilk

Only two parallel programming primitives:

1. (binary) **fork**, and
2. (binary) **join.**



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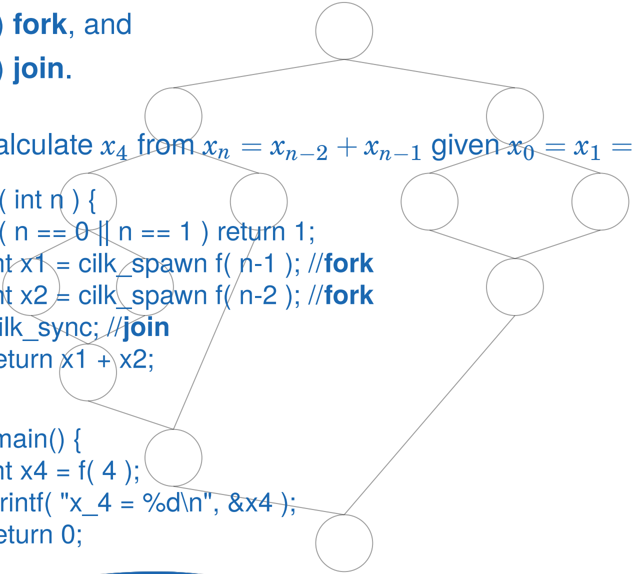

Example: calculate x_4 from $x_n = x_{n-2} + x_{n-1}$ given $x_0 = x_1 = 1$:

```
int f( int n ) {
    if( n == 0 || n == 1 ) return 1;
    int x1 = cilk_spawn f( n-1 ); //fork
    int x2 = cilk_spawn f( n-2 ); //fork
    cilk_sync; //join
    return x1 + x2;
}

int main() {
    int x4 = f( 4 );
    printf( "x_4 = %d\n", &x4 );
    return 0;
}
```

Cilk

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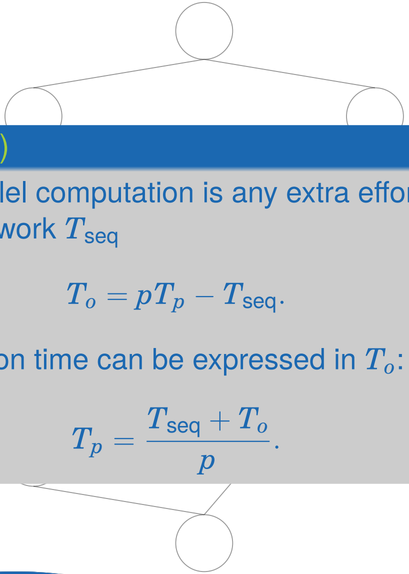

Definition (Overhead)

The **overhead** of parallel computation is any extra effort expended over the original amount of work T_{seq}

$$T_o = pT_p - T_{\text{seq}}.$$

The parallel computation time can be expressed in T_o:

$$T_p = \frac{T_{\text{seq}} + T_o}{p}.$$

Cilk

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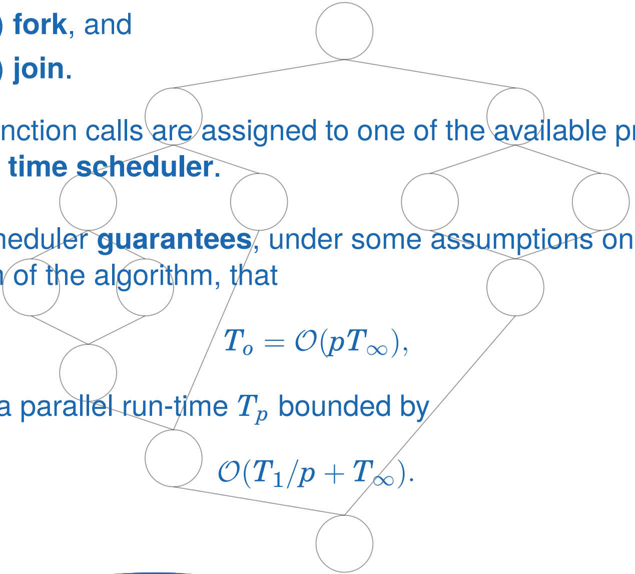

Spawned function calls are assigned to one of the available processes by the Cilk **run time scheduler**.

The Cilk scheduler **guarantees**, under some assumptions on the determinism of the algorithm, that

$$T_o = \mathcal{O}(pT_\infty),$$

resulting in a parallel run-time T_p bounded by

$$\mathcal{O}(T_1/p + T_\infty).$$

MapReduce

Google Pregel is a successor of **MapReduce**, a parallel framework that operates on **large data sets** of key-value pairs

$$S \subseteq K \times V,$$

with K a set of possible **keys** and V a set of **values.**

MapReduce defines two operations on S:

$$\text{map: } K \times V \to K \times V;$$

$$\text{reduce}(k)\text{: } \mathcal{P}(\{k\} \times V) \to K \times V, \quad k \in K.$$

- The map operation is **embarrasingly parallel**: every key-value pair is mapped to a new key-value pair, an entirely local operation.

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– The map operation is **embarrasingly parallel**: every key-value pair is mapped to a new key-value pair, an entirely local operation.
– The reduction reduces **all** pairs in S that have the same key k, into **one** single key-value pair: **global communication.**

MapReduce

Calculating $\alpha = x^T y$ using MapReduce:

$$\text{Let } S = \{(0, \{x_0, y_0\}), (1, \{x_1, y_1\}), \ldots\}.$$

1. Map: for each pair $(i, \{a, b\})$ write $(\text{partial}, a \cdot b)$. Applying this map adds $\{(\text{partial}, x_0 y_0), (\text{partial}, x_1 y_1), \ldots\}$ to S.
2. Reduce: for all pairs with key ‘partial’, combine their values by addition and store the result using key α. Applying this reduction adds $(\alpha, \sum_{i=0}^{n-1} x_i y_i)$ to S.
3. Done: S contains a single entry with key α and value $x^T y$.

The set S is safely stored on a **resilient file system** to cope with hardware failures.

Pregel

Consider a graph $G = (V, E)$. **Graph algorithms** may be phrased in an SPMD fashion as follows:

- For each vertex $v \in V$, a thread executes a user-defined SPMD algorithm;
- each algorithm consists of successive local compute phases and global communication phases;
- during a communication phase, a vertex v can only send messages to $N(v)$, where $N(v)$ is the set of neighbouring vertices of v; i.e., $N(v) = \{w \in V \mid \{v, w\} \in E\}$.

MapReduce and Pregel are variants of the BSP algorithm model!

- a type of **fine-grained BSP**.
- parallelism in Pregel is slightly odd to think about;
  e.g., what does its compute graph look like?

Fine-grained summary

Optimisation targets and performance metrics:

- Optimise algorithms to maximise **parallelism** and (thus) minimise the **algorithmic span**;
- run-time scheduler with **bounded overhead** (e.g., pT_∞ for Cilk).

Questions:

1. does this account for all realistic overheads, in your experience?
2. does more parallelism always mean better performance?
3. we wrote Cilk bounded the parallel run-time by $\mathcal{O}(T_1/p + T_\infty)$. Is there a difference between T_{seq} and T_1?

Fine-grained summary

Answers:

- *Q: does this account for all realistic overheads, in your experience?*
- A: **no.** Not accounted for are: memory overhead, and, most importantly, the costs of **data movement**!

Definition (Memory overhead)

Let M_{seq} be the memory requirement of a sequential algorithm, and let M_p be the memory requirement of a parallel algorithm that solves the same problem. Then the parallel overhead in memory is

$$M_o = pM_p - M_{\text{seq}}, \text{ or, rewritten:}$$

$$M_p = \frac{M_{\text{seq}} + M_o}{p}.$$

(Cilk bounds M_o, whereas other fine-grained schemes may not.)

Fine-grained summary

Answers:

- *Q: does more parallelism always mean better performance?*
- A: **No**; for starters, we typically have less than ∞ processors. Also: maximum speedup is relative to the chosen algorithm; other algorithms that solve the same problem may be preferable!

Example:

Consider the naive $\Theta(n^2)$ Fourier transformation; its span is $\Theta(\log n)$, so its parallelism is $\Theta(n^2/\log n)$. **Lots of parallelism**!

The FFT formulation has work $\Theta(n \log n)$, also with span $\Theta(\log n)$ resulting in $\Theta(\frac{n \log n}{\log n}) = \Theta(n)$ parallelism. **Less parallelism...**

Question: how many processors do you need to achieve a theoretical parallel processing time equal to the span, for both the naive and the fast algorithm?

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Question: how many processors do you need to achieve a theoretical parallel processing time equal to the span, for both the naive and the fast algorithm? Answer: naive $\Theta(n^2)$ processors, FFT $\Theta(n)$ processors.

Fine-grained summary

Answers:

- *Q: is there a difference between considering T_{seq} or T_1?*
- A: **definitely**; there may be multiple sequential algorithms to solve the same problem. When comparing, always **compare to the best**. For parallel sorting:

$$S^{\text{odd-even sort}} = T_{\text{seq}}^{\textbf{qsort}} / T_p^{\text{odd-even sort}}.$$

In some cases, however, using T_1 does make sense; for instance, when measuring T_o to see if the implementation behaves as expected from a theoretical analysis:

$$T_o^{\text{odd-even sort}} = pT_p^{\text{odd-even sort}} - T_{\text{seq}}^{\textbf{odd-even sort}}.$$

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Questions

Regarding the Cilk parallelisation of $x_n = x_{n-1} + x_{n-2}, x_0 = x_1 = 1$:

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}
```

Questions:

- what is T_1 (asymptotically)?
- what is T_∞ (asymptotically)?
- what is T_{seq}?
- is this trivial parallelisation a good idea?

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```

- $T_1 = \Theta(2^n)$, $T_\infty = \Theta(n)$, but
- $T_{\text{seq}} = n$! This parallelisation makes no sense.

Questions

In the bulk synchronous parallel setting, how do the concepts such as span and overhead translate?

Question:

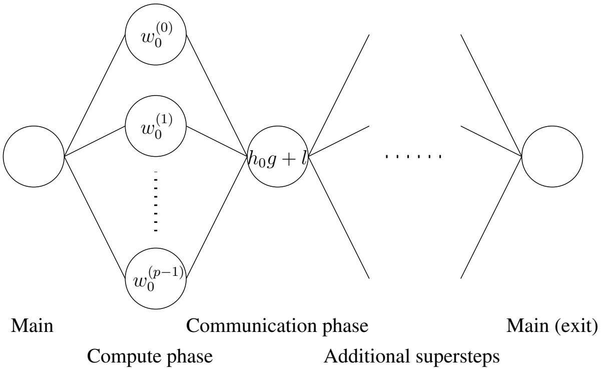
- how do the computation graph, T_∞, and T_o look for BSP?

BSP and scalability

Q: what is the BSP computation graph, span, and overhead?

Graph:

- Sequential part, SPMD (supersteps), sequential part.
- The SPMD part is coarse-grained over p processors.



BSP and scalability

Q: what is the BSP computation graph, span, and overhead?

Span:

- Critical path follows compute phases with maximum work $w_i^{(s)}$, and
- communication phases with cost proportional to h_i;
- $T_\infty = T_p = \sum_{i=0}^{N-1}(\max_s w_i^{(s)} + h_i g + l)$.
- The span is the BSP cost!

BSP and scalability

Q: what is the BSP computation graph, span, and overhead?

Overhead (as in $T_p = \frac{T_{\textsf{seq}} + T_o}{p}$):

$$T_o = p \left(\sum_{i=0}^{N-1} \max_s w_i^{(s)} + h_i g + l \right) - T_{\textsf{seq}}.$$

Data movement, latency costs, and extra computations on top of the bare minimum required, all add up to the overhead.

BSP and scalability

Definition (Efficiency)

Given T_{seq}, p, and T_p, the **efficiency** E of the parallel algorithm is

$$E = S/p = \frac{T_{\text{seq}}}{pT_p}.$$

An algorithm is **scalable** (in time) if E is constant as $p, T_{\text{seq}} \to \infty$.

Question:

- Express E in terms of T_{seq} and T_o

BSP and scalability

Express E in terms of T_{seq} and T_o, answer:

$$E^{-1} = \frac{pT_p - T_{\text{seq}}}{T_{\text{seq}}} + 1 = \frac{T_o}{T_{\text{seq}}} + 1.$$

The efficiency only depends on T_{seq} and T_o:

$$E(T_{\text{seq}}, p) = \frac{1}{1 + \frac{T_o(T_{\text{seq}}, T_p)}{T_{\text{seq}}}}.$$

An algorithm is scalable if T_o/T_{seq} is kept constant. This principle is known as **iso-efficiency** (as advocated by Grama et al.).

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Questions: how is iso-efficiency affected when considering

1. strong scalability, i.e., $S = \frac{T_{\text{seq}}}{T_p} = \Omega(p)$ as $p \to \infty$ while T_{seq} is assumed constant.
2. weak scalability, i.e., $S = \mathcal{O}(1)$ as $T_{\text{seq}} \to \infty$ while p is constant.

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Answers:

1. strong scalability:
$$E \sim S/p = \frac{\Omega(p)}{p} = \Omega(1).$$
2. weak scalability,
$$E \sim \frac{\mathcal{O}(1)}{p} = \mathcal{O}(1).$$

That is, both **strong and weak scalability induce iso-efficiency**.

BSP and scalability

Question: Is a BSP algorithm with $T_p = \mathcal{O}(T_{\text{seq}}/p + p)$ strongly scalable? How about weakly? What is its iso-efficiency?

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I.e., when $T_p = \mathcal{O}(T_{\text{seq}}/p + p)$, there is no strong scalability.

Question: for a small constant c, do we have strong scalability if

1. $T_p = T_{\text{seq}}/p + cT_{\text{seq}}$,
2. $T_p = T_{\text{seq}}/p + c$,
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Answers:

1. No: $T_o = cpT_{\text{seq}}$,
2. No: $T_o = cp$,
3. Yes: $T_o = c$; strong scalability means constant overhead.

BSP and scalability

For non-embarrasingly parallel applications, **strong scalability does not exist.** But is that a bad thing?

Definition (Amdahl's Paradox)

Given an algorithm with a serial part of size c and a parallel part of size $(1-c)$, then the maximum speedup of that algorithm is given by

$$S = \frac{T_{\text{seq}}}{T_{\text{seq}}(c+(1-c)/p)} = \frac{1}{1/p - c/p + c}.$$

Note that for $p \to \infty$, $S \to \frac{1}{c}$, 'and therefore parallel computing is of extremely limited use' (paraphrased from Amdahl).

If $c = 10^{-2}$ (one percent), the maximum speedup is 100. For $c = 10^{-6}$, $S \leq 10^6$, etc.

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If $c = 10^{-2}$ (one percent), the maximum speedup is 100. For $c = 10^{-6}$, $S \leq 10^6$, etc. So to scale, we need **a lot more** parallelisable work than non-parallelisable work. 'This naturally happens when $T_{\text{seq}} \to \infty$' (paraphrased from Gustavson); this is what led to **weak scalability**.

BSP and scalability

Recall that for BSP algorithms:

$$T_o = p\left(\sum_{i=0}^{N-1} \max_s w_i^{(s)} + h_i g + l\right) - T_{\text{seq}}.$$

Iso-efficiency then requires that the following expression be kept constant:

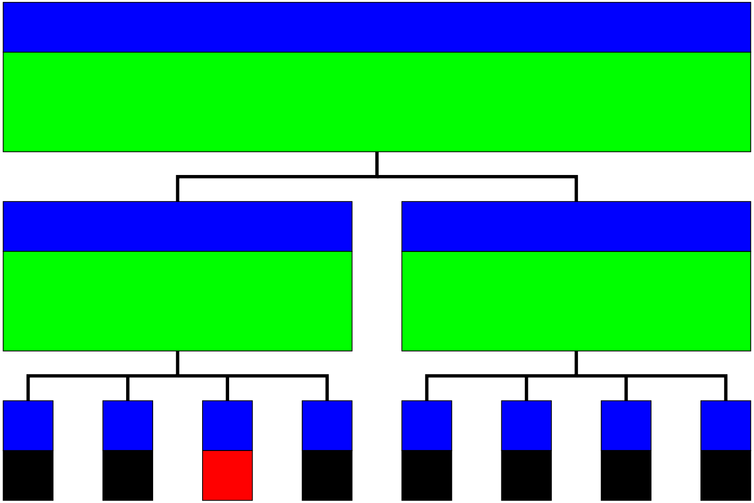
$$E^{-1} = 1 + T_o/T_{\text{seq}} = p\left(\sum_{i=0}^{N-1} \frac{\max_s w_i^{(s)} + h_i g + l}{T_{\text{seq}}}\right).$$

For BSP, usually $T_p > T_{\text{seq}}/p + g + l$; we never expect strong scalability; but like always, making sure that $T_p \sim T_{\text{seq}}$ gets us weak scalability.

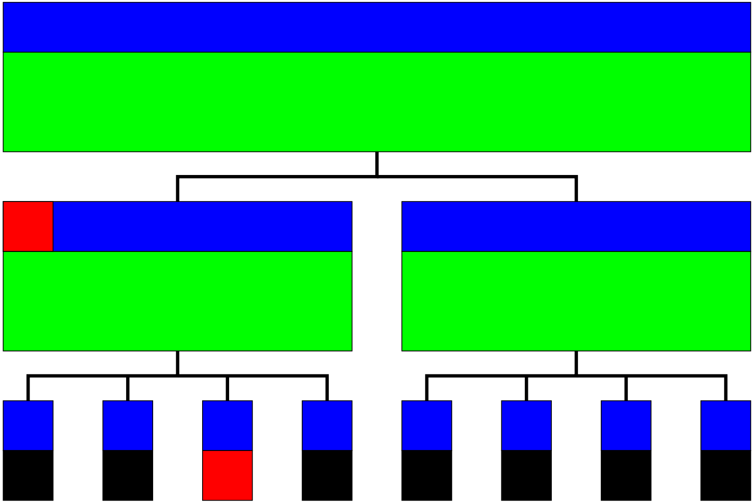
Algorithm design

ExaScience life lab

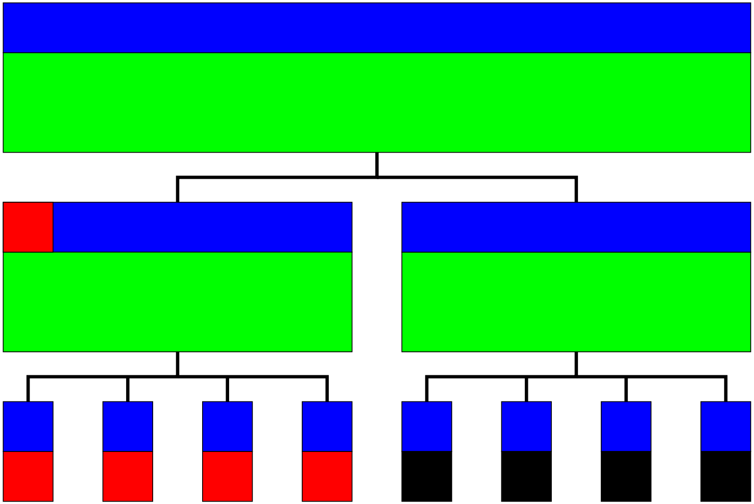
Multi-BSP: broadcasting



Multi-BSP: broadcasting



Multi-BSP: broadcasting



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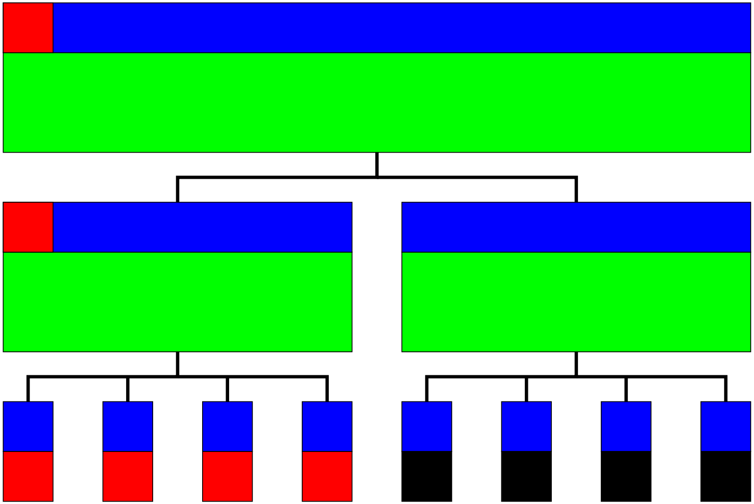

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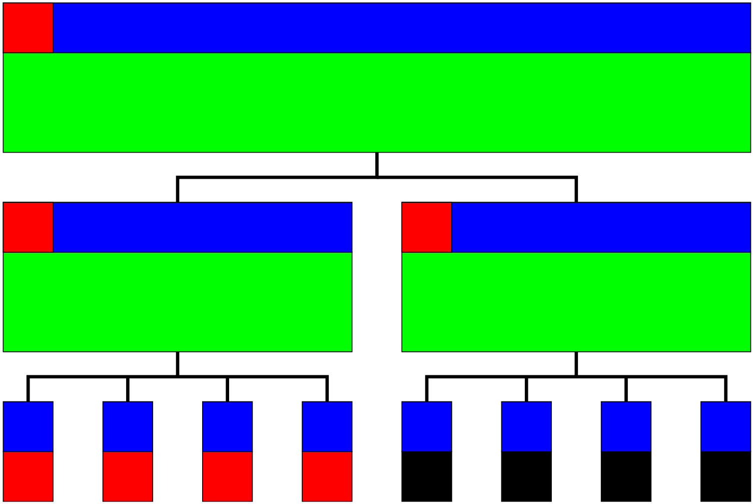
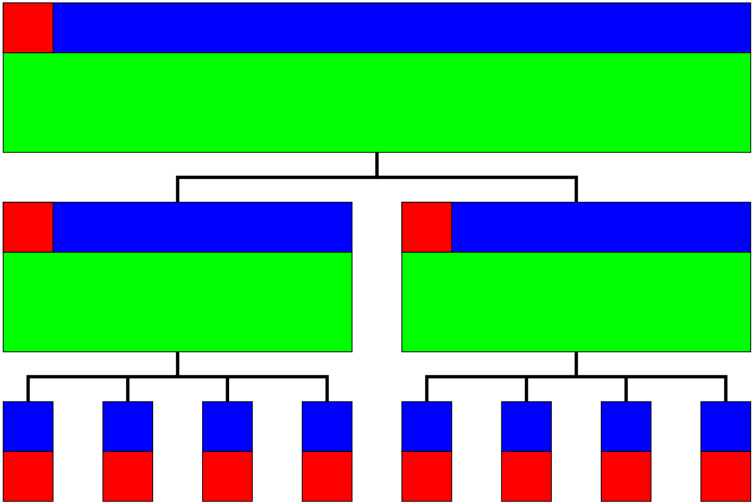


Multi-BSP: broadcasting



Multi-BSP: broadcasting

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Due to the imposed Multi-BSP computer model, each node **must** be visited, even in this minimal example.

For any non-trivial Multi-BSP algorithm,

$$T_o \geq 2p \sum_{k=0}^{L-1} (g_k + l_k).$$

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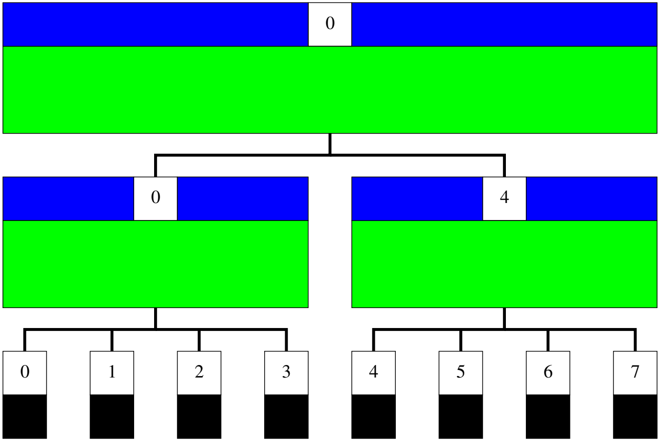
For any non-trivial Multi-BSP algorithm,

$$T_o \geq 2p \sum_{k=0}^{L-1} (g_k + l_k).$$

Question: is **storing** the broadcast value $2p - 1$ times mandatory?

Multi-BSP: broadcasting

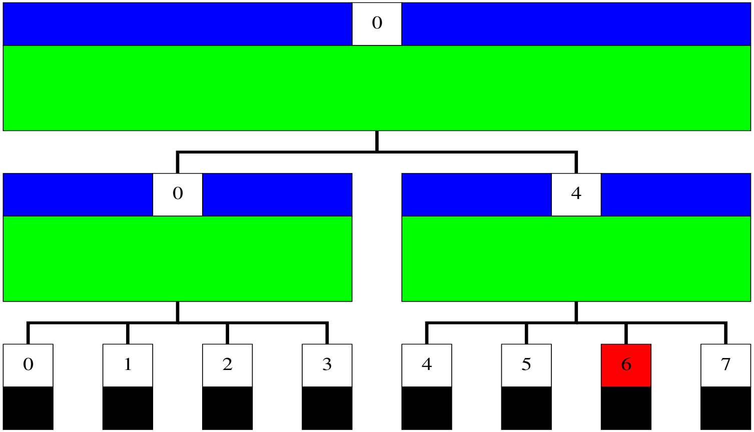
Memory embedding (shared address space):



Memory requirements are now bounded from below as $M_p \geq M_{\text{seq}}/p + p$.

Multi-BSP: broadcasting

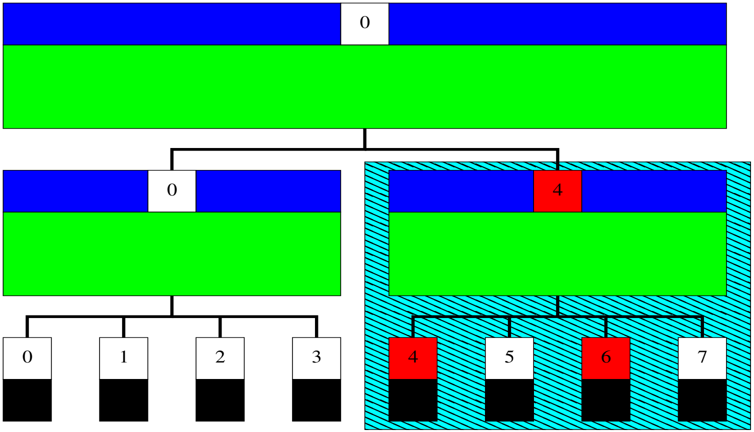
Optimal algorithm with embedding (shared address space):



Start SPMD section, entry at leaf level, leaf 6 is source.

Multi-BSP: broadcasting

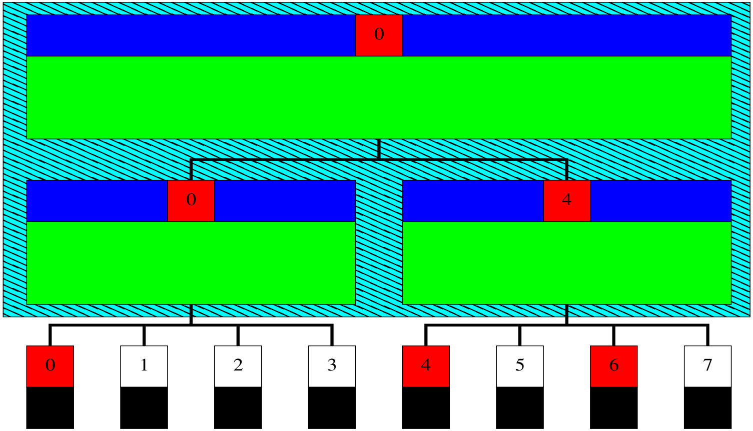
Optimal algorithm with embedding (shared address space):



On this local BSP computer, communicate val to PID 0 and **move up**.

Multi-BSP: broadcasting

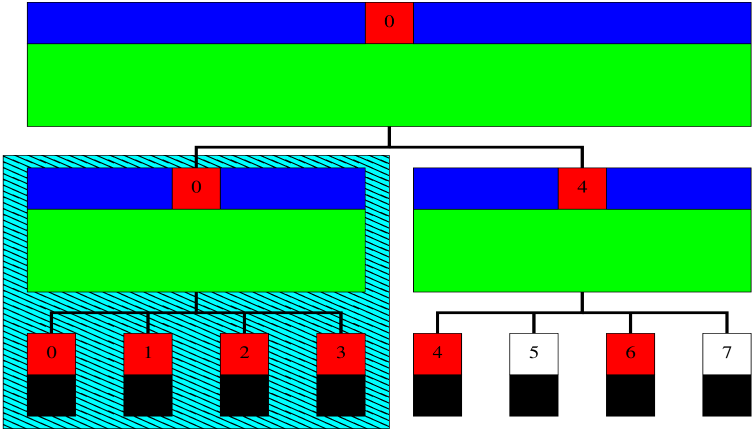
Optimal algorithm with embedding (shared address space):



On this upper level, send val to PID 0, **broadcast**, and **move down.**

Multi-BSP: broadcasting

Optimal algorithm with embedding (shared address space):



On this level, only PID 0 has val; **broadcast**, and done.

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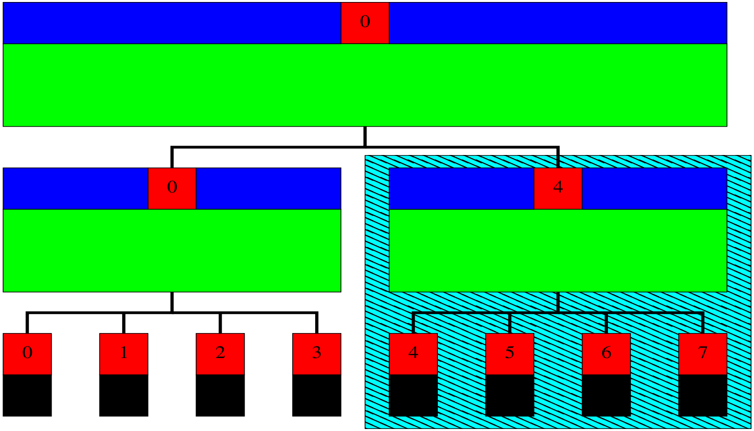

On this level, only PID 0 has val; **broadcast**, and done.

Multi-BSP: broadcasting

Input: val (a value or $\emptyset$),
Output: the value that was broadcast.

```
if $val \neq \emptyset$ then
    set $source = true$
else
    set $source = false$
while not on the Multi-BSP root do
    if $source$ and $bsp\_pid() \neq 0$ then
        send $val$ to PID $0$
    move upwards in the Multi-BSP tree
while not on a leaf node do
    if $bsp\_pid() = 0$ then
        for $k = 1$ to $bsp\_nprocs()$ do
            send $val$ to PID $k$
    move downwards in the Multi-BSP tree
return $val$
```

New Multi-BSP primitives

To control the flow up and down the Multi-BSP tree:

- `bsp_up()`, and
- `bsp_down()`.

These are **synchronising** primitives, like `bsp_sync()`.

To inspect Multi-BSP tree information:

- `bsp_lid()`, the leaf ID of this SPMD program;
- `bsp_leaf()`, whether this SPMD program runs on a leaf;
- `bsp_sleaves()`, the number of leaf nodes in this subtree;
- `bsp_nleaves()`, the total number of leaf nodes in the full tree.

That's it!

Multi-BSP summary

- Non-trivial Multi-BSP algorithms require that

$$T_p = \Omega(T_{\text{seq}}/p + \sum_{k=0}^{L-1}(g_k + l_k));$$

If each node in the tree runs an SPMD program and is non-trivial, then

- leaf nodes distribute the sequential work,
- internal nodes communicate at least one word and synchronise once while going up the Multi-BSP computer tree, and
- synchronise again while going down the tree.

The minimal non-trivial parallel cost thus is as given above.

Multi-BSP summary

– non-trivial Multi-BSP algorithms require that

$$pM_p = \Omega(M_{\text{seq}} + p), \text{ i.e., } M_o = \Omega(p);$$

Each Multi-BSP node is represented in memory, together with at least the M_{seq} data required for the entire problem, from which the above follows.

Multi-BSP summary

  – communication can still be done using ‘horizontal’ primitives.

Algorithm data does not need to be replicated by using **memory embedding**. This also enables **horizontal communication**, just like what we are used to from ‘flat’ BSP.

Multi-BSP summary

- in general, **no compute** on **non-leaf** nodes.

In principle, the leaf node with PID 0 could handle the computation of its parent Multi-BSP node. Higher-level nodes can choose a representative in a recursive fashion, similar to memory embedding.

Distributing part of the computational cost over non-leaf nodes in this fashion, necessarily uses less than p processors. Doing so leaves the remaining processes idle, thus contributing unnecessary overhead.

Multi-BSP summary

– we have demonstrated a Multi-BSP broadcasting algorithm with cost

$$T_p^{\text{multibsp-bcast}} = \sum_{k=0}^{L-1} \left(g_k + l_k + T_k^{\text{bsp-bcast}} \right), \text{ with}$$

$$T_k^{\text{bsp-bcast}} = \min_{b \in \{2,\ldots,p\}} ((b-1)g_k + l_k) \log_b p_k .$$

The Multi-BSP broadcast traverses the Multi-BSP tree a minimal number of times; only once up, and once down.

– we can still use known BSP algorithms within Multi-BSP algorithms;

– only a few additional primitives enable writing portable codes.

Multi-BSP summary

- Non-trivial Multi-BSP algorithms require that

$$T_p = \Omega(T_{\mathsf{seq}}/p + \sum_{k=0}^{L-1}(g_k + l_k));$$

- non-trivial Multi-BSP algorithms require that

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- communication can still be done using ‘horizontal’ primitives;
- in general, **no compute** on non-leaf nodes;
- we have demonstrated a Multi-BSP broadcasting algorithm and determined its Multi-BSP cost;
- we can still use known BSP algorithms within Multi-BSP algorithms;
- only a few additional primitives enable writing portable codes.